Bayesian Methods for Measuring and Managing Operational Risks

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Outline

- **Introduction: Regulatory Developments**
- **Bayes Rule**
  - Prior Beliefs, Sample Likelihood and Posterior Density
- **Bayesian Networks**
  - Architecture, Back Testing, Scenario Analysis, Decision Networks
  - *Examples: Settlement Loss, Number of Fails, Human Risks*
- **Bayesian Estimation**
  - Estimation as a Decision
  - *Example : Estimating a Loss Probability (PE) using Internal and External data*

1. Introduction

Regulatory Timetable

- Sep 01 - CP 2 ½ Discussion document on advanced approaches
- Jan 02 - Basle CP3 '*Op-Risk Standards'*
- End 02 - Basle II Final document (and EU)
- 03 - 04 - Adaptation of national rules
- Jan 05 - Implementation among G-10

Operational Risks

- IN
  - Fraud, theft, unauthorised activities
  - Transaction and other human errors
  - Legal, regulatory and compliance failures
  - Systems failures
  - Acts of god
- OUT
  - Business risks (strategic)
  - Reputational risk
  - Cost of controls

Stages for Operational Risk Models

- A four stage approach aims to make capital charges progressively lower and more risk sensitive

1. Single indicator
2. Standardised lines of business (LOB)
3. Internal Risk Based (IRB)
4. Loss distribution approach (LDA)

- Aimed at flexibility, as opposed to 'one size fits all', but qualifying criteria become increasing stringent

Stage 1: Single Indicator

- One operational risk indicator, such as **gross income**, for the entire bank
- Capital charge will be alpha% of this basic indicator
- This approach concords with the view that risk capital to cover 'other risks' should be whatever it takes to top up the total risk capital in the system, after credit charges have been changed by the implementation of internal models in Basel II
- CP2 mentioned a figure of 20% of capital allocation for Op-Risk and estimates (which would translate to an alpha of about 30% of annual gross income) but this is now thought to be too high.

Stage 2: Standardized LOB

Bank	**Risk Indicator** / **Rate**
LOB_1	Gross Income / β_1
LOB_2	Average Assets / β_2
..	
..	
LOB_n	Total Funds / β_n

Stage 3: IRB Approach

Bank	**Risk Type 1**	**....**	**Risk Type k**
LOB_1	EI / γ_{11}	.....	EI / γ_{1k}
LOB_2	EI / γ_{21}	.....	EI / γ_{2k}
..		..	
..		..	
LOB_n	EI / γ_{n1}	....	EI / γ_{nk}

Exposure Indictors ?

	LOB (at level 1)	**Exposure Indictor**
Investment Banking	Corporate Finance	Volume of new deals
	Trading and Sales	Volume of trades
	Retail	Volume of transactions
Banking	Commercial	Volume of transactions
	Payment and Settlement	Volume of transactions
	Agency Services	Value of assets in custody
Other	Asset Management	Value of assets under management
	Retail Brokerage	Value of transactions

Operational Risk Requirement

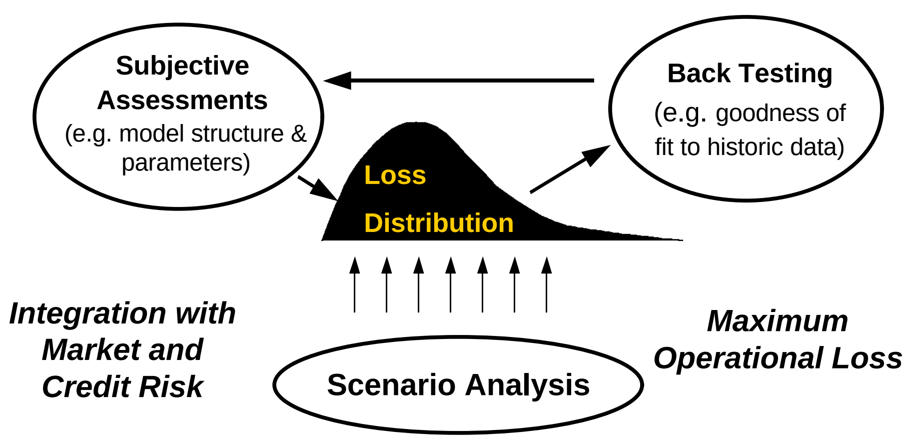
- Given estimates of the probability of a loss event (PE) and the expected loss given event (LGE):
  - Expected loss = Exposure Indictor x PE x LGE;
  - Capital charge = Expected Loss x gamma
  - Total ORR is the sum of separate charges
- This approach assumes standard deviation is a multiple of expected loss, as in the binomial model
- Note that the exposure indicator should be in frequency rather than volume terms (*Binomial Gammas,* Operational Risk, April 2001). Then, assuming variability σ in loss severity, for a 99% confidence level

$$\text{gamma} \approx 7 \sqrt{[(1 + (\sigma/\text{LGE})^2)/(\text{EI} \times \text{PE})]}$$

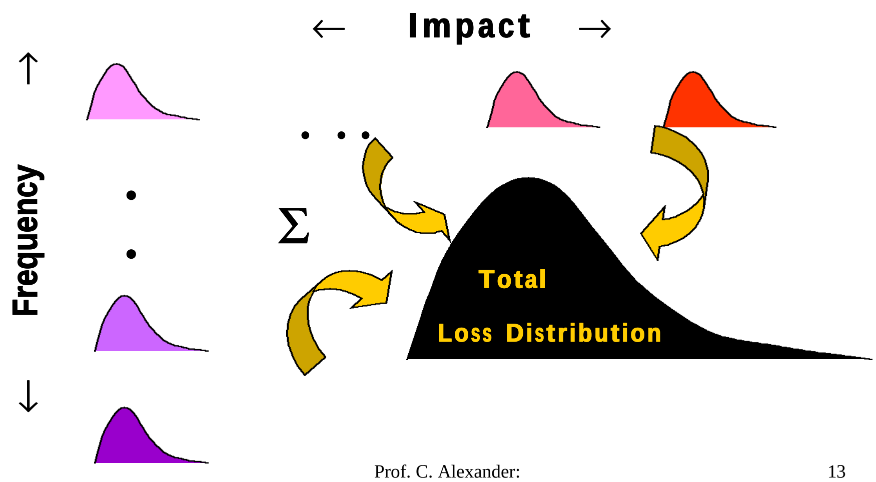
Risk Profile Index

- Regulators may provide industry wide gamma estimates, but allow firms greater flexibility to use their own gamma estimates
- The RPI is the ratio of internal to external gamma estimates
- Qualifying criteria: banks need rigorous processes for estimating exposure indictor, PE and LGE based on several years of data
- Bayesian estimation is an attractive approach to the use of external data consortiums

Stage 4: Internal Models



Frequency and Impact



Low Frequency – High Impact

- Include Fraud and Acts of God
- These risks
  - Can jeopardize the ability of the firm to function
  - Should have relatively high gammas (Stage 3 models)
  - Very little internal data $\Rightarrow$ Bayesian Estimation
  - Classical statistical approach: Use EVT to fit impact and Poisson to fit frequency (Stage 4 models)

High Frequency – Low Impact

- Include human risks and transactions processing risk
- These risks
  - Do not normally jeopardize the ability of the firm to function but do influence the value of the firm
  - Contribute more to the expected total operational loss, rather than the tail loss
  - Require control procedures rather than large risk capital requirements
  - Should have relatively low gammas (Stage 3 models)
  - Plenty of data $\Rightarrow$ Bayesian belief networks (BBNs)
    - Loss distributions (Stage 4 models)
    - Key performance indicators (Management of Op-Risk)

2. Bayes Rule

The **Reverend Thomas Bayes** was born in London (1702) and died in Kent (1761).

His **Essay Towards Solving a Problem in the Doctrine of Chances,** published posthumously in 1763, laid the foundations for modern statistical inference.

Subjective Choice and Prior Beliefs

- Subjective choice is much more of an issue in operational risk than it is in market or credit risk measurement:
  - Inadequacy of the data
  - Uncertainty in the modelling process
- Subjective choice is incorporated in a Bayesian model as **prior beliefs** about the values of **model parameters.**

Classical vs Bayesian Methods

Classical

Op Risk applications include EVT

Assume that at any point in time there is a 'true' value for a model parameter.

Bayesian

Op Risk applications include BBNs

What is the probability of the model parameter given the data?

Bayes' Rule

- For two events X and Y, their joint probability is the product of the conditional probability and the unconditional probability:

$$\text{Prob}(X \text{ and } Y) = \text{prob}(X|Y)\ \text{prob}(Y)$$

- Or, by symmetry:

$$\text{Prob}(X \text{ and } Y) = \text{prob}(Y|X)\ \text{prob}(X)$$

$$\text{prob}(X|Y) = [\ \text{prob}(Y|X) / \text{prob}(Y)]\ \text{prob}(X)$$

Interpretation of Bayes' Rule

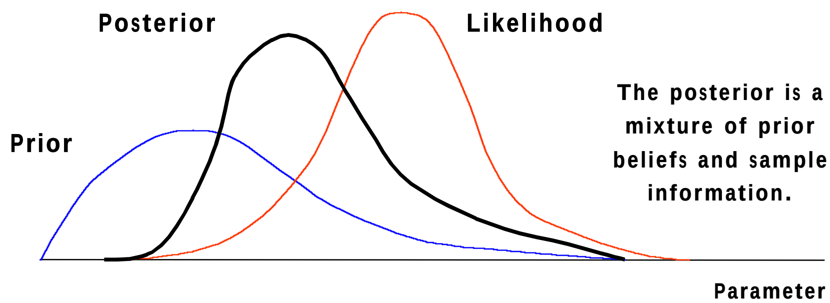
prob(parameters|data) = prob(data|parameters)*prob(parameters) / prob(data)

Posterior Density	$\propto$	**Sample Likelihood**	*	**Prior Density**

$$f_{\theta|X}(\theta|X) \quad \propto \quad f_{X|\theta}(X|\theta) \ * \ f_{\theta}(\theta)$$

This is how Bayesian models allow prior beliefs about the value of a parameter, which may be very subjective, to influence parameter estimates.

Posterior Density $\propto$ Sample Likelihood * Prior Density



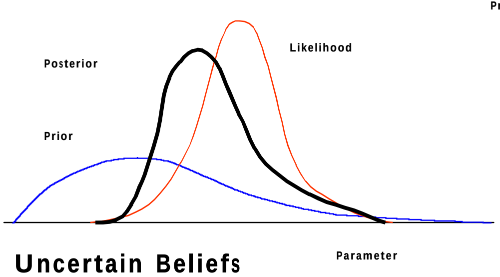
The posterior is a mixture of prior beliefs and sample information.

No prior information (uniform prior) $\Rightarrow$ posterior $\equiv$ likelihood

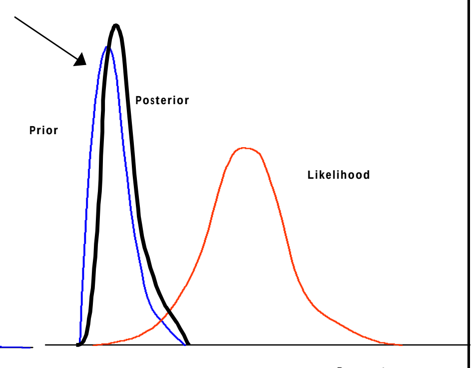
No sample information (uniform likelihood) $\Rightarrow$ posterior $\equiv$ prior

The Effect of Prior Beliefs

If prior beliefs are expressed with a great deal of certainty, Bayesian estimates will be close to prior expectations *and they will have small standard errors*



Uncertain Beliefs



Confident Beliefs

3. Bayesian Networks (BBNs)

Advantages:

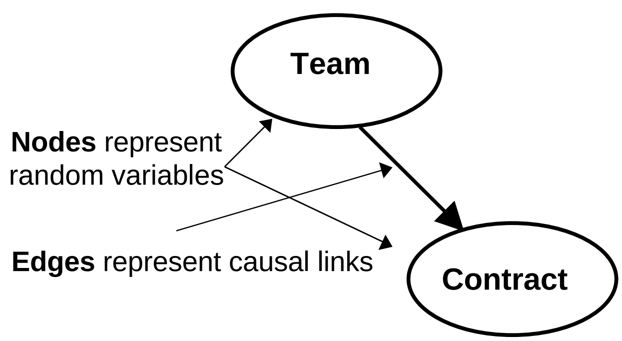
- BBNs describe the factors that are thought to influence operational risk, thus providing explicit incentives for behavioural modifications;
- They provide a framework for scenario analysis: to measure maximum operational loss, and to integrate operational risk with market and credit risk;
- Augmenting a BBN with decision nodes and utilities improves transparency for management decisions. Thus decisions may be based on 'what if?' scenarios

Limitations:

- No unique structure; a BBN is a picture of the mind of the modeller. Therefore BBNs require much clarity in their construction and rigorous back testing

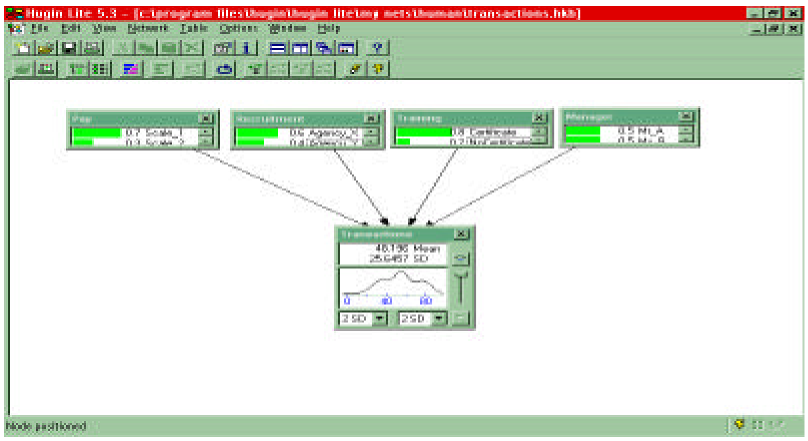
Architecture

The basic structure of a Bayesian network is a directed acyclic graph



Note: Amongst others, Wilson (1999), Alexander (2000, 2001) and King (2001) have advocated the use of BBNs for modelling high frequency low impact operational risks.

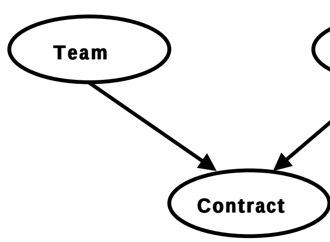
Discrete and Continuous Nodes



Bayes Rule

Prob(good) = 0.75

Prob(bad) = 0.25



Prob(good) = 0.5

Prob(bad) = 0.5

Prob(lose) = 0.3125

Prob(win) = 0.6875

Prob(lose) = prob(lose | T = good and M = good) prob(T = good and M = good)

+ prob(lose | T = good and M = bad) prob(T = good and M = bad)

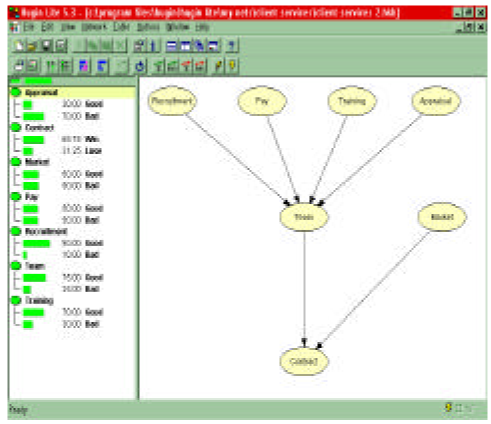
+ prob(lose | T = bad and M = good) prob(T = bad and M = good)

+ prob(lose | T = bad and M = bad) prob(T = bad and M = bad)

Describing the Network

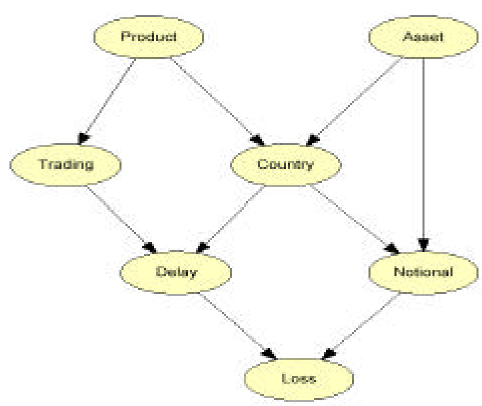
Nodes, edges, and probabilities are added to model the influence of causal factors for each node

The Bayesian network is completed when all initial nodes can be assigned probabilities



Example: Settlement Loss

Operational (as opposed to credit) settlement loss is "**the interest lost and the fines imposed as a result of incorrect settlement**"



Initial Probabilities

Asset	
80.00	FX
20.00	Security

Country	
47.20	Europe
52.80	Asia

Product	
30.00	Underlying
70.00	Derivative

Trading	
17.00	OTC
83.00	Exchange

Delay	
85.65	None
6.18	1 day
3.63	2 days
1.94	3 days
0.88	4 days
1.72	> 4 days

Notional	
13.96	<10
10.00	10-20
10.36	20-30
17.64	30-40
26.36	40-50
21.68	>50

Loss	
90.22	0
3.11	0-1,000
1.77	1,000-2,000
1.50	2,000-3,000
1.40	3,000-4,000
1.31	4,000-5,000
0.69	5,000-10,000

Expected Loss = 239.3$

99% Tail Loss = 6,750$

(per transaction)

Scenario Analysis:
Maximum Operational Loss

Asset	
* 100.00	FX
-	Security

Country	
-	Europe
* 100.00	Asia

Product	
-	Underlying
* 100.00	Derivative

Trading	
* 100.00	OTC
-	Exchange

Delay	
50.00	None
30.00	1 day
10.00	2 days
1.00	3 days
1.00	4 days
8.00	> 4 days

Notional	
10.00	<10
5.00	10-20
5.00	20-30
25.00	30-40
30.00	40-50
25.00	>50

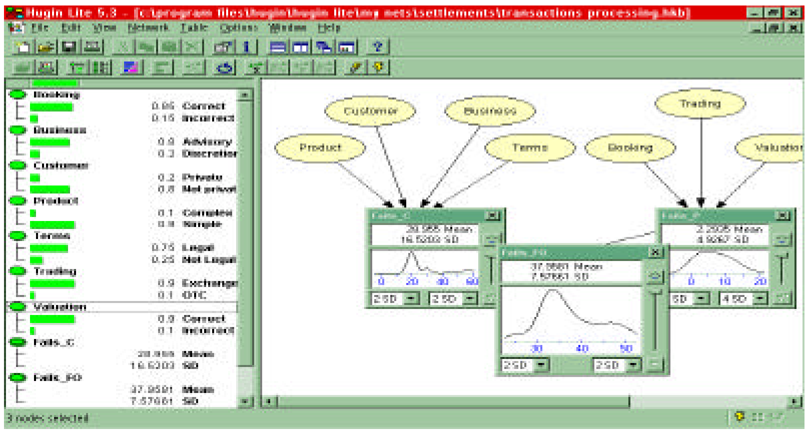
Loss	
64.70	0
9.50	0-1,000
6.02	1,000-2,000
5.61	2,000-3,000
5.39	3,000-4,000
5.66	4,000-5,000
3.12	5,000-10,000

Expected Loss = 957.7$

99% Tail Loss = 8,400$

(per transaction)

Example: Number of Fails



BBNs for Human Risks

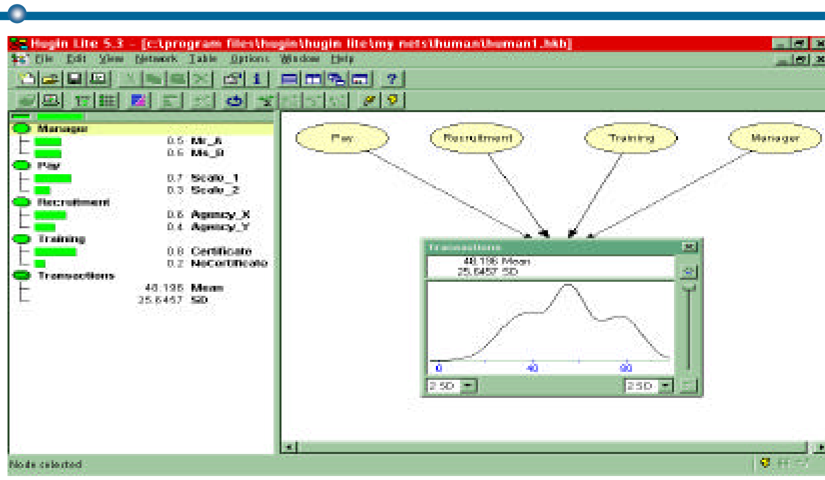
Human risk has been defined as the risk of **inadequate** staffing for required activities

- **Measures of human adequacy:**
  - Key Performance Indicators
  - Balanced Scorecard (Kaplan & Norton)
- **'Causal' factors or 'Attributes':**
  - Lack of training
  - Poor recruitment processes
  - Loss of key employees
  - Poor management
  - Working culture

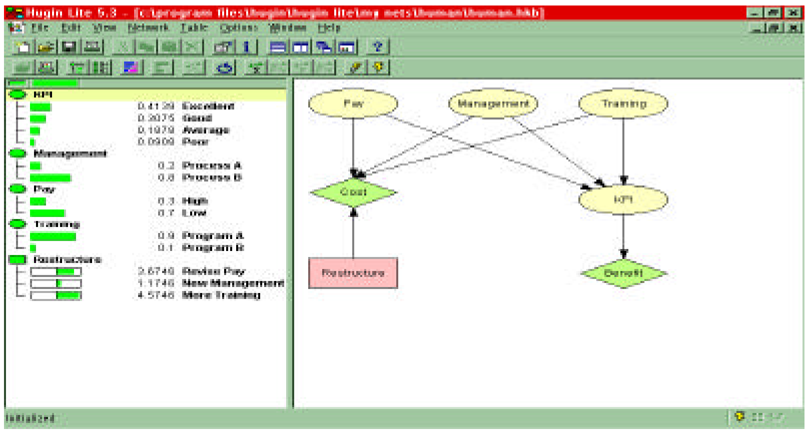
Key Performance Indicators

Function	Quantity	Quality
Back Office	Number of transactions processed per day	Proportion of internal errors in transactions processing
Middle Office	Timeliness of reports Delay in systems implementation; IT response time	Proportion of errors in reports Systems downtime
Front Office	Propriety traders: 'Information ratio' Sales: Number of contacts	Proportion of ticketing errors; Time stamp delays Credit quality of contacts; Customer complaints

Example:
Number of Transactions Processed



Bayesian Decision Networks



4. Bayesian Estimation

- A Bayesian regards the process of estimation as a decision:

Prior density → Posterior density
Likelihood of current data → Posterior density
Posterior density → Expected loss
Loss function → Expected loss
Expected loss → Choose the parameter to minimize expected loss

Bayesian Estimators

Standard loss functions:		*Optimal estimator:*
Zero-One	→	**Mode of posterior**
Absolute	→	**Median of posterior**
Quadratic	→	**Mean of posterior**

Maximum likelihood estimation (MLE) is a crude form of Bayesian estimation

Example

Stage 3 Models: Capital charge = γ x EI x **PE** x LGE

- Consider the problem of estimating the probability of a write-down loss due to fraudulent or unauthorized activity or theft.
- Very little internal data $\Rightarrow$ The MLE of PE will be inaccurate
- The right hand column gives the operational risk capital for this loss type assuming EI = 50m\$, LGE = 1m\$ and γ = 10

Year:	**# Deals**	**# losses**	**PE**	**ORR (m$)**
Year 1	500	2	0.004	2
Year 2	400	3	0.0075	3.75
Year 3	600	3	0.005	2.5

Bayesian Estimates for Loss Probabilities

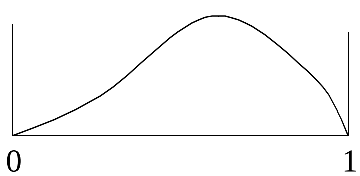
- Now let us suppose that **external data** on 10,000 deals are available and 30 of these incurred loss due to fraudulent or unauthorized activity or theft.
- If this is used as prior information for Bayesian estimates of the the loss probability in each year, the PE and ORR estimates are:

Year	PE	ORR
1	0.002953	1.4765
2	0.003078	1.539
3	0.003019	1.5095

- These Bayesian estimates are close to the prior mean because of the large size of the external data set.
- A smaller (less confident) external data set ⇒ parameter estimates will be more of a mixture between internal and external data.

Beta Densities

- More or less any views on the value of a loss probability, p may be approximated by a beta density
  $f(p) \propto p^a (1-p)^b$ $\quad 0 < p < 1.$



- Regard both internal data (sample likelihood) and external data (prior beliefs) as beta densities: In the previous example:

$$\textbf{prior} \propto p^{30}(1 - p)^{9970}$$

$$\textbf{year 1 likelihood:} \propto p^{2}(1 - p)^{498}$$

- Therefore year 1 posterior is another beta density:

$$p^{32}(1 - p)^{1468}$$

Mean of Beta Density

- Suppose the loss function is quadratic, so the Bayesian estimator is the mean of the posterior
- Suppose the year n posterior is the beta density:

$$\mathbf{p^a(1 - p)^b}$$

- That includes both internal and external data.
- Using the fact that

$$\int p^x (1-p)^y \, dp = x!\, y! \,/(x+y+1)!$$

it follows that the mean of this beta density, that is the Bayesian estimate, is

$$\mathbf{(a+1)/(a+b+2)}$$

Summary

- **Bayesian belief networks** have many applications to modelling high frequency low impact operational risks, where the focus should be on improved risk management and control procedures.
- A Bayesian network **improves transparency** and their facility for **scenario analysis** leads to more efficient risk management;
  - E.g. 'maximum operational loss' scenarios can be identified to help the operational risk manager focus on the important factors that influence operational risk.
- Management of operational risks may be facilitated by the use of **Bayesian decision networks**, which allow decision makers to base choices on 'what if?' scenarios.

Summary

- Regulators hope to develop an industry-wide loss distribution for each **loss type** (write-down, loss or damage to assets, legal liability etc) and each **line of business** (corporate finance, trading and sales, asset management, etc)
- A **Risk Profile Index** (RPI) will allow individual firms to modify their capital charge if their internal loss distribution is very different to the industry norm
- So how should a firm estimate PE and LGE? **Bayesian estimation** methods are useful when the available internal data are sparse (e,g, for low frequency-high impact risks) or when internal processes have been changed

Selected Further Reading

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Useful Links: Performance Measures

- hrba.org (Human Resources Benchmarking Association) and fsbba.org (Financial Services and Banking Benchmarking Association)
- afit.af.mil and pr.doe.gov/bsc001.htm (Balance Scorecard meta-resource pages)
- bscol.com (Balance Scorecard Collaborative - Kaplan and Norton) and pr.doe.gov/pmmfinal.pdf (Guide to Balance Scorecard Methodology)
- mentorme.com/html/D-Keyperfind.html and totalmetrics.com/tr-kpa.htm (Monitoring KPIs)
- kpisystems.com/case_studies/banking/bi_kpi_ops_values.htm (some KPIs for banking operations)

Useful Links: Bayesian Networks

- http.cs.berkeley.edu/~murphyk/Bayes/bnsoft.html (list of free Bayesian network software)
- dia.uned.es/~fjdiez/bayes (meta-resource page for Bayesian networks)
- research.microsoft.com/research/dtg/msbn/default.htm (*MSBN* a free non-commercial Excel compatible BBN)
- hugin.dk (leading commercial BBN with free demo version *Hugin Light*)
- lumina.com (makers of *Analytica*, leading software package for quantitative business models)
- dcs.qmw.ac.uk/research/radar (Risk Assessment and Decision Analysis Research, QMW College London and their consultancy agena.co.uk specializing in risk management of computer-based systems)
- genoauk.com (Operational risk consultancy firm)
- algorithmics.com (Watchdog Bayesian network product)
- eoy.co.uk (Ermst and Young Bayesian network product)