ISDA Counterparty Risk
Notes on “α”

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Overview

- **Introduction**

- **Trading Book Issues**
    - Current Regulatory Capital Treatment
    - Basel II Proposal
    - Issues

- **ISDA Proposal**
    - Expected Positive Exposure (EPE)
    - “α”

- **Questions**

Trading Book Issues

Trading Book: Current Treatment

What's in the trading book?

- Positions meeting the following criteria:
    - Actively managed positions, usually only held short-term
    - Market and credit risk positions can be traded on a "recognised exchange or representative market"
    - Positions are marked to market daily
- OTC derivatives, Repos, securities lending and borrowing ("SLB"), prime brokerage
- Also unsettled securities purchases and sales
- Inventory – trading book risk only (market risk / specific risk), not discussed here

Current capital requirements

- Market risk capital for all net market and specific risk positions – "1996 Amendment"
- Credit risk capital for counterparty risk (risk of counterparty non performance)
    - for everything *except* inventory;
    - Inventory is treated via specific risk.
- Counterparty risk charge is a "banking book" type charge – similar levels and methodology to loan credit risk, although the underlying transactions are in the trading book:

Capital = Loan Equivalent Exposure × 2 - 8%

Example

- OTC Derivatives: **Capital requirement = (CMV + Addon) × [4%]**

Trading Book: Basel II Proposal

Basel II/CAD III capital requirements

- 2 – 8% gets replaced by Standardized or IRB risk weight of counterparty as you'd expect

Capital = LEE × Standardised/IRB RW × 8%

- Changes to the LEE for repos and SLB, possibly for unsettled transactions, not (yet) for OTC
- Market risk capital calculation does not change

Example

- Repos, IRB Approach: **Capital requirement = (MV +VaR) × IRB RW × 8%**
- Allowed to use internal VaR models to calculate LEE but only if accompanied by onerous backtesting and multipliers

Key Trading Book Issues I

Consistent treatments

- Current rules are very different for repos and OTC derivatives, and also differ by jurisdiction
- Basel II changes repo treatment, but not OTC - now both frameworks are complex, but still very different
- SEC proposals seem to treat repo and OTC the same, but differently from Basel I *and* Basel II
- So the current situation is a mess – much more uniform treatment must be desirable

Use of internal models

- Key to industry's concerns has been the exposure or "loan equivalent" measure for capital calculation

Capital = Loan Equivalent × IRB risk weight × 8%

- Industry wants to be allowed to use internal calculations of their loan equivalents
- Note this is not the same as being allowed to calculate capital directly – no chance of that!
- Basel II does (rather reluctantly) allow this for repo – style transactions, but not for OTC derivatives

EPE vs PFE vs VaR

- What measure should be taken as the exposure for capital purposes – PFE, EPE or VaR?
- ISDA-LIBA-TBMA etc says EPE, and we've supported this with extensive work
- For repos, Basel II currently requires VaR, a much more conservative measure than EPE
- For OTC, the current addons were calibrated in a complex way – sort of PFE but not exactly

Key Trading Book Issues II

Maturity

- Special issues for repos (how to treat short dated exposures?) and OTC (does long maturity affect risk?)
- Current Basel II proposals provide some capital mitigation for short dated transactions
- ISDA proposal for repos would lead to more beneficial treatment
- But has been hard to find consensus on this issue

Scope – settling transactions

- Basel II proposals for repo-style transactions might also apply to other securities transactions such as "settling transactions" i.e. unsettled securities purchases and sales
- This could create big implementation problems for banks – and Basel have not made up their minds yet

Other

- Validation standards
  - Basel II proposals allow VaR but only accompanied by onerous backtesting and multipliers
  - Validation standards also need developing for OTC. Industry still wants less onerous standards
- Also settling transactions, CDS counterparty risk, joint default risk…

Review of trading book issues – Joint IOSCO / Basel Committee Working Group

- On 15 January, the Basel Committee announced a working group to review all these issues
- Strongly welcomed by the industry – but naturally, uncertainty remains until conclusions are known

ISDA Proposal

Dialogue on Trading Book

		Event/publication	Counterparty Risk	Securities financings
1988	July	BIS - Basel I	Addons for derivatives, still in use.	Credit equivalent = market value
1995	April	"Treatment of potential exposure for off balance sheet items"	Aggregation rule and extended addons	
2000	October	ISDA WG formed.	Review capital treatment of counterparty risk / propose new rules	
2001	January	CP2	PFE mentioned in passing as exposure measure for derivatives	"w" factor and haircuts
	May	ISDA response to CP2	New counterparty risk rules based on EPE	"w" factor criticised
	July	Letter Counterparty Risk SG to ISDA	Raised issues: (1) weak independence (2) wrong way exposure (3) time horizon (4) validation. (5) EPE "not industry practice"	-
	August	Letter ISDA to Models Task Force	Addressed "not industry practice" reservation (5)	Similarity between repo and counterparty risk.
	September	BIS publish update	- w factor eliminated	- w factor eliminated ' - Possible use of VaR for repos instead of HCs
	October	Letter ISDA-LIBA-TBMA to CRM SG	-	EPE indicative levels for repos.
2002	January	Letter from MTF to ISDA	Commitment to review counterparty risk when Basel II is done	
	April	Letter from CRM Subgroup to ISDA/LIBA/TBMA	-	VaR for repos subject to suitable suggestions for backtesting and validation
	May	Reply ISDA-LIBA-TBMA to CRM Subgroup	-	Suggest backtesting framework for VaR (at request of CRM subgroup)
	June	ISDA-LIBA-TBMA & CRM SG	-	VaR for repos to be allowed - but subject to backtesting / multipliers
	July	Response to ISDA's proposals (May 02) by M Gibson, Federal Reserve	Detailed technical response to ISDA's CP2 (May 01) proposal -New analysis of weak independence and maturity.	-
2003	March	ISDA counterparty market practice survey	ISDA surveys member firms for information on margin and collateral and actual portfolios' correlation characteristics.	-
	March	Letter ISDA-LIBA-TBMA to CRM SG	- Call for review of trading book instruments - Create common approach to repos and OTC derivatives (based on EPE)	
	April	CP3	-	Backtesting and "punitive" multipliers
	June	ISDA/LIBA/TBMA publish revised proposals	- Report of member firms' survey. -"α" proposal with detailed simulations and analytic work	- To be regarded as margined derivatives - same value for α
	July	ISDA response to CP3		- Proposal for treatment of short maturities. - Criticism of multipliers
	October	Basel Committee Press Release - EL/UL "Madrid" Proposals	Questions about EL treatment for counterparty risk	-
	November	SEC Proposals for Broker - Dealers	Does not appear to distinguish OTC derivs from repos - all referred to as "derivatives"	Proposes VaR calculation (similar to Basel II)
2004	January	Communication from CRM WG		Reduced multipliers as per ISDA suggestion
	January	Basel Committee press release - "Continued Progress toward Basel II"	- Trading book review announced to cover counterparty risk and repos - To be undertaken by a joint Basel - IOSCO working group (hence SEC will be involved)	

The Maturity Issue

Maturity in the IRB approach

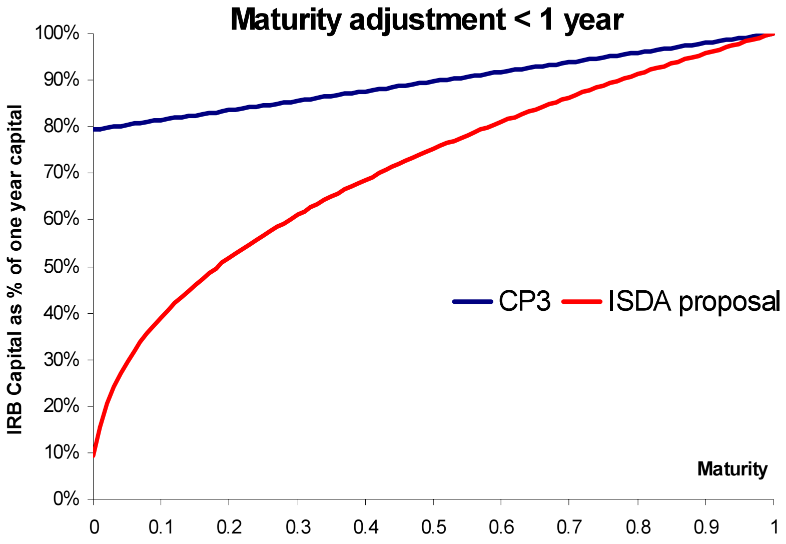
- IRB risk weight incorporates a maturity dimension, designed with loans in mind
- Application to repos (short dated) and OTC derivs (longer dated) is problematic

Repos – ISDA proposals

- Any concession for short – dated maturity would be material for repos
- CP3 proposals include small concessions for maturities < 3 months, by simply using the existing “maturity adjustment” (CP3 para 292)
- ISDA CP3 response has a somewhat more radical proposal (as shown)

OTC derivatives

- Often *long* dated – opposite case to repos
- No very strong evidence supporting any maturity adjustment for OTC’s
- This is because value of OTC derivatives does not respond sensitively to counterparty credit spread
- This view is broadly mirrored in the Federal Reserve response to ISDA proposals
- However there may be a practical need for a maturity dimension, willing to consider this



OTC Derivatives Rules

Current rules

$$\text{Capital requirement} = \overbrace{(\text{CMV} + \text{Addon})}^{\text{“Loan Equivalent Exposure”}} \times [4\%]$$

- Addon determined from a table published by the BIS in 1988.
- “Aggregation rule” applies to calculate addon in presence of netting. Not conceptually sound.

ISDA 2001

$$\text{Capital requirement} = \text{EPE} \times \text{IRB risk weight} \times 8\%$$

- EPE (expected positive exposure) replaces MV + Addon
- EPE is a VaR – like measure, but more suited to capital calculations.
- Proposal can be summarised as: Make EPE the Loan Equivalent Exposure.

ISDA 2003

$$\text{Capital requirement} = \alpha \times \text{EPE} \times \text{IRB risk weight} \times 8\%$$

- A multiplier α is needed to fully cover risks.
- The work described here helped ISDA to quantify α in a range **$100\% \leq \alpha \leq 120\%$.**

Repos Rules

Current rules

$$\text{Capital requirement} = \text{Exposure} \times [2 - 8\%]$$

- “Exposure” measure is anything from replacement cost to full notional amount in various local regulatory regimes.

Basel II – CP2, 2002

$$\text{Capital requirement} = (\text{MV} + \Sigma(E_S \times H_S)) \times \text{IRB RW} \times 8\%$$

- or if permitted to use counterparty level VaR

$$\text{Capital requirement} = (\text{MV} + \text{VaR}) \times \text{IRB RW} \times 8\%$$

ISDA 2003

$$\text{Capital requirement} = \alpha \times \text{EPE} \times \text{IRB RW} \times 8\%$$

- α same value as for OTC derivatives

Expected Positive Exposure

Definition of Expected Positive Exposure (EPE)

- For a given portfolio at a given time, exposure is the maximum of 0 and market value
- Actual exposure forms a path over time, so the time average (over 1 year) can be taken, but this depends on the paths taken by the market variables
- Average of this over market paths can then be taken ⇒ Expected Positive Exposure

$$E(t) = \max(0, V(t))$$

$$EPE = \mathbb{E}(\frac{1}{T}\int_{t=0}^{T}\max(0, V(t))dt)$$

Why is EPE the right measure?

- The IRB framework measures systematic risk = default risk due to economic change
- Under certain conditions, this means that only the average amount of exposure present is relevant – to use PFE (at a high percentile) is "double counting"

Alpha

The need for alpha

- We have proposed EPE as the right measure of exposure for derivatives
- But EPE only quantifies the systematic element of risk - the concentration risk of derivatives is bigger than for loans with the same EPE

Definition of alpha

- α quantifies the extra capital needed for derivatives, relative to loans with same EPE
- Equivalent information to "understatement" $U = \alpha - 1$ (used by Fed)

$$\alpha = \frac{\text{Actual Capital}}{\text{Capital per ISDA 2001 proposal}}$$

Quantifying alpha

- Work done by both Fed and ISDA to quantify α using:
    - Actual firms' portfolios (calculations performed by member firms)
    - Stylised portfolios, with parameters determined from real data (Eduardo Canabarro)
    - Analytic results (Michael Gibson, Washington Fed and Tom Wilde)

Conclusions on alpha

- We found **$\alpha \sim 1.09$** ($U \sim 9\%$) in most portfolios, but α can be larger in extreme cases
- ISDA's new proposals suggest a range as the basis for further discussion (ISDA, July 03):

$$1.1 \leq \alpha \leq 1.2$$

Alpha Results – Firms' Own Calculations

Firms' own calculations

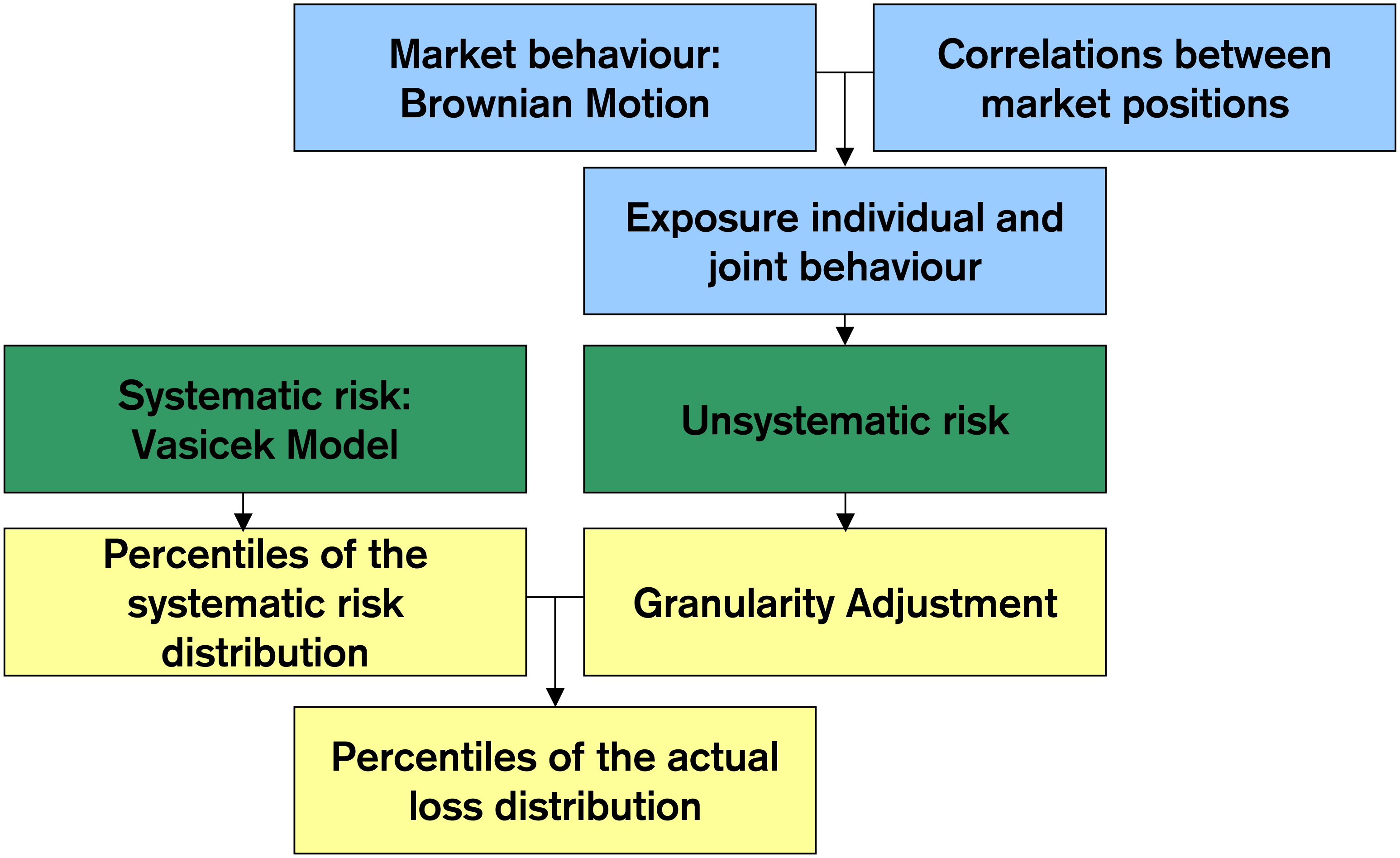
- Members of the ISDA working group estimated alpha for their firms' internal derivatives portfolios, using the ISDA definition of alpha.
- This is possible for firms having a simulation or other approach to capital which captures the additional risks measured by alpha.
- Results in good agreement with each other and with work on sample portfolios (see below).

Portfolio	α
Example portfolio	**1.09**
Firm 1	1.08
Firm 2	1.10
Firm 3	1.07
Firm 4	1.07

Analytic α – Methodology Overview

Steps to analytic calculation of loss – counterparty risk portfolio

- Main ingredients: Vasicek model for credit risk, Brownian motion for market risk
- Special model needed for correlations between counterparties
- Risks are put together using the granularity adjustment method
- Monte – Carlo work done by Eduardo Canabarro showed good agreement



Example Portfolio

Example portfolio

- This portfolio was constructed by Eduardo Canabarro, on suggestions of Evan Picoult
- Intended to be a realistic model of a large derivatives portfolio.
- Portfolio parameters were as shown for “base” portfolio.

PORTFOLIO PARAMETERS

Percentile	q	99.90%
Credit information		
Number of counterparties	N	200
Default probability	p	0.30%
Asset correlation	l	22.00%
Exposure information		
Number of market factors	K	3
Spot values	$\pm u$	± 1.36
Market volatilities	σ	1 unit

Within IRB range (12% to 24%)

K = 3 chosen based on the empirical correlation structure discussed above

Results

Asset corr'n	Spot value+/-	No factors	No. of cpties	PD	Conf level	*Systematic risk*	Actual Portfolio A		Reference Portfolio B		α = A/B	
λ	u	K	N	p	q	*Analytic*	M-Carlo	Analytic	M-Carlo	Analytic	M-Carlo	Analytic
Base case						*Percentile*	*Percentile*		*Percentile*			
22%	1.36	3	200	0.3%	99.9%	*10.19*	**13.14**	**12.96**	**12.06**	**12.02**	**1.09**	**1.08**
Sensitivity to asset correlation												
0%	1.36	3	200	0.3%	99.9%	*0.51*	6.09	NA	4.26	NA	1.43	1.46
12%	1.36	3	200	0.3%	99.9%	*5.31*	8.99	8.91	7.43	7.73	1.21	1.15
24%	1.36	3	200	0.3%	99.9%	*11.30*	14.08	13.96	13.04	13.05	1.08	1.07
50%	1.36	3	200	0.3%	99.9%	*30.69*	32.70	32.50	32.06	31.82	1.02	1.02
Sensitivity to current market values												
22%	**0**	3	200	0.3%	99.9%	*5.65*	8.42	8.23	6.24	6.18	1.35	1.33
22%	**1**	3	200	0.3%	99.9%	*8.26*	10.96	10.81	9.61	9.61	1.14	1.12
22%	**2**	3	200	0.3%	99.9%	*14.28*	17.80	17.64	16.95	16.96	1.05	1.04
22%	**3**	3	200	0.3%	99.9%	*21.24*	25.95	25.73	25.19	25.26	1.03	1.02
Sensitivity to the number of market risk factors												
22%	1.36	**1**	200	0.3%	99.9%	*10.19*	13.22	13.11	12.02	12.02	1.10	1.09
22%	1.36	**5**	200	0.3%	99.9%	*10.19*	13.07	12.93	12.10	12.02	1.08	1.08
22%	1.36	**10**	200	0.3%	99.9%	*10.19*	12.97	12.91	12.01	12.02	1.08	1.07
22%	1.36	**50**	200	0.3%	99.9%	*10.19*	12.96	12.89	12.00	12.02	1.08	1.07
Sensitivity to number of counterparties												
22%	1.36	3	**20**	0.3%	99.9%	*1.02*	3.54	3.72	2.81	2.85	1.26	1.31
22%	1.36	3	**50**	0.3%	99.9%	*2.55*	5.21	5.26	4.27	4.37	1.22	1.20
22%	1.36	3	**100**	0.3%	99.9%	*5.10*	7.79	7.83	7.08	6.92	1.10	1.13
22%	1.36	3	**500**	0.3%	99.9%	*25.48*	28.92	28.36	27.81	27.31	1.04	1.04
Sensitivity to probability of default												
22%	1.36	3	200	**0.1%**	99.9%	*4.55*	7.03	6.93	6.01	6.16	1.17	1.12
22%	1.36	3	200	**0.5%**	99.9%	*14.56*	17.59	17.56	16.44	16.50	1.07	1.06
22%	1.36	3	200	**1.0%**	99.9%	*23.10*	26.60	26.50	25.09	25.20	1.06	1.05
22%	1.36	3	200	**5.0%**	99.9%	*59.40*	65.00	64.55	61.90	61.84	1.05	1.04
Sensitivity to confidence level												
22%	1.36	3	200	0.3%	**99.0%**	*4.37*	6.08	6.11	5.68	5.56	1.07	1.10
22%	1.36	3	200	0.3%	**99.5%**	*5.85*	7.90	7.90	7.18	7.23	1.10	1.09

Actual portfolio A

- This is the portfolio with full stochastic exposures and correlations as per the settings described above

Reference portfolio B

- This portfolio is as A but with each exposure fixed = EPE of the corresponding portfolio A counterparty

α = A/B

- alpha measures the extra risk arising from the fact that exposures are variable and correlated

Agreement

- The analytic results agree well to MC

Summary

Summary

Issues with proposed treatment of Trading Book under Basel II

- Need for a consistent treatment for trading book exposures
- Treatment should allow banks to use internal exposure measures – but which measure? And at what cost?
- How to adjust for short/long maturities for repos/OTC derivatives?

ISDA's proposal

- Exposure measure
  - Same treatment for Repos/SLB as for OTC derivatives
  - Based on IRB approach but using EPE as a measure of Loan Equivalent Exposure
  - Multiplier ("α") to take into account greater unsystematic risks and pair-wise correlations due to market factors
- Maturity adjustment
  - More favourable treatment for repos (short-term)
  - Not penalising long-dated OTC derivatives (no evidence that an adjustment is needed)

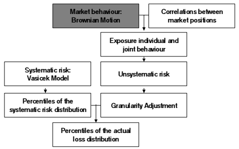
Next Steps

- Review of trading book issues – Joint IOSCO / Basel Committee Working Group
- Generally encouraging although uncertainty remains until conclusions are known

Questions

Annex – Detail on ISDA's α methodology

Exposure Mean and Variance



Input parameters

- Need input market volatility and spot market value
- Convenient to work with dimensionless parameters

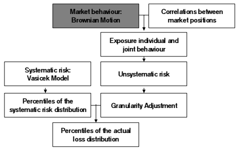
Input Parameter	**$**	**Dimensionless**
Annualised volatility	ω_A	1 by definition
Spot value	$V_A(0)$	$u_A = V_A(0) / \omega_A$

Expected Positive Exposure and Root Mean Square Exposure

- The key exposure measures arising in this analysis.
- Throughout, we assume that counterparty values $V_A(1)$ are normally distributed.

Definitions	
Expected Exposure (EPE)	$E_A = \mathbb{E}(\max(0, V_A(1)))$
Root mean square exposure (RMSE)	$F_A^2 = \mathbb{E}(\max(0, V_A(1))^2)$
Formulae	
Expected Exposure (EPE)	$E_A / \omega_A = u_A N(u_A) + n(u_A)$
Root mean square exposure (RMSE)	$(F_A / \omega_A)^2 = (u_A^2 + 1) N(u_A) + u_A n(u_A)$

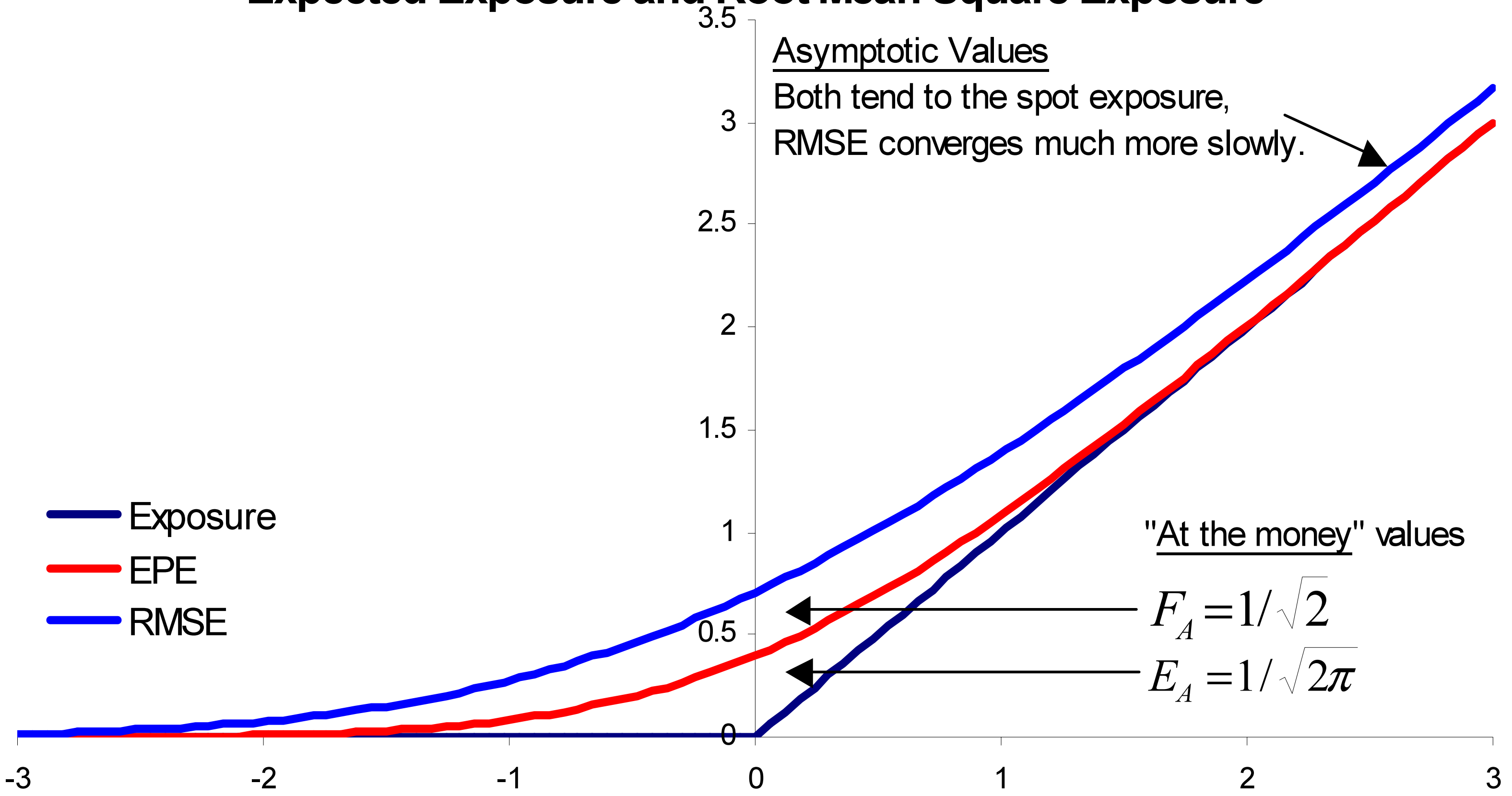
EPE and Root Mean Square Exposure



EPE and RMSE are both similar to call option price

- Both look like option on Exposure = Max(0, V(1)).
- EPE actually same as “arithmetic Brownian motion” option valuation
- Would be Black Scholes if we assumed geometric Brownian motion.

Expected Exposure and Root Mean Square Exposure



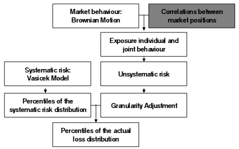
Asymptotic Values

Both tend to the spot exposure, RMSE converges much more slowly.

"At the money" values

$F_A = 1/\sqrt{2}$

$E_A = 1/\sqrt{2\pi}$



Exposure Covariance

EPE and RMSE only describe individual counterparty portfolios

- Also need to understand pairwise exposure covariance between counterparties:

$$c_{AB} = \mathbb{E}(\max(V_A(1),0)\max(V_B(1),0)) - E_A E_B$$

- This is driven by market correlation ρ = Corr($V_A(1), V_B(1)$).
- Positive exposure correlation is a kind of concentration risk.
- Similar type formula to EPE and RMSE (to second order in the market correlation ρ

$$c_{AB} \cong \omega_A \omega_B (\rho N(u_A) N(u_B) + \frac{\rho^2}{2} n(u_A) n(u_B))$$

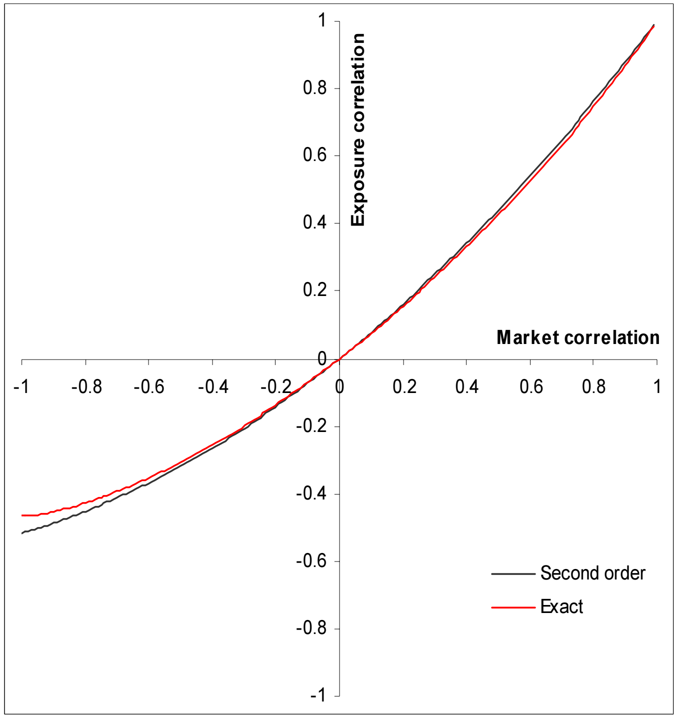
Correlation Graph

- Graph shows the correlation implied by c_{AB} (i.e. $c_{AB}/\sqrt{(c_A c_B)}$ for $u_A = u_B = 0$.
- The graph also shows the exact result which is available in closed form for $u_A = u_B = 0$.

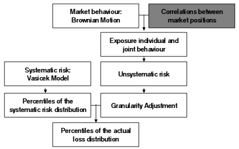
Notes

- Exposure correlation is clearly a convex function of value correlation. e.g.:

$$\rho_{Exposure}(+1) = 1 \quad \rho_{Exposure}(-1) = -1/(\pi - 1) \cong -0.46$$

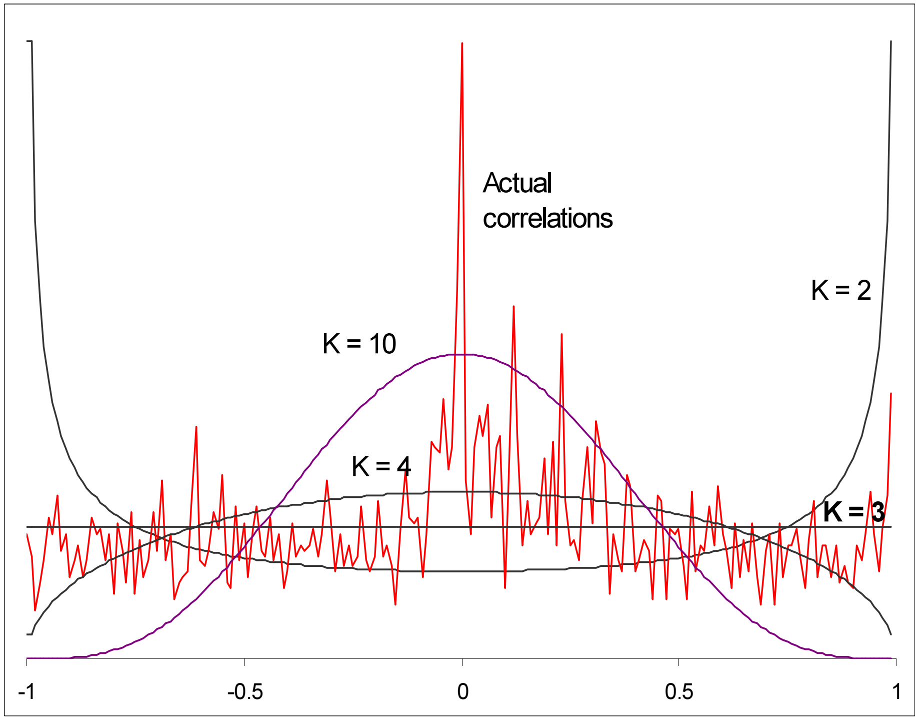


Market Correlations – Empirical Evidence



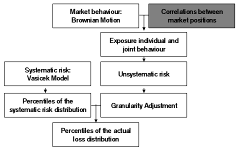
Counterparties may be independent, but only “on average”

- Any pair of counterparties selected at random, will tend to have non – zero correlation
- Due to coincidental common positions in markets, and correlation between markets
- Usually tends to average out to zero across the portfolio – as shown (**red line**)



This correlation structure gives rise to added risk.

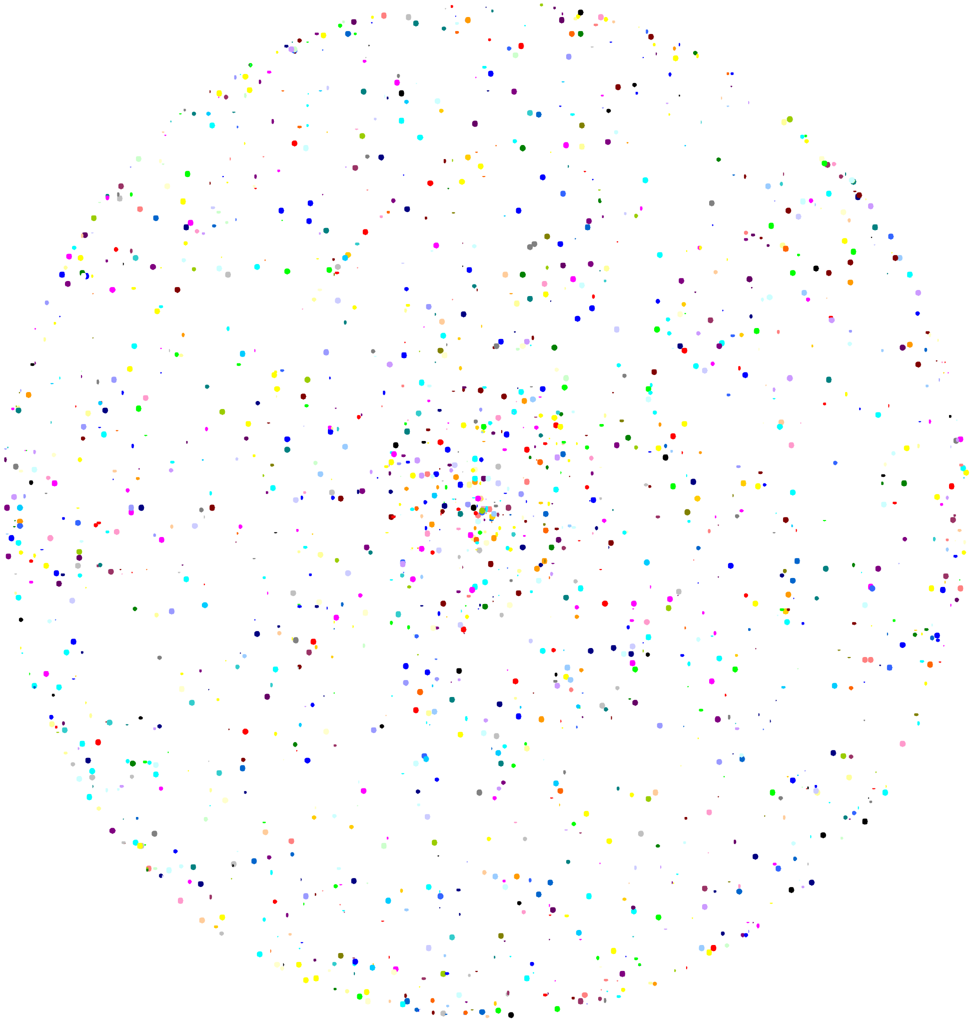
- Due to convexity of the exposure correlation, scattered market correlations imply positive average exposure correlation.



Correlation Model

Model correlations as scattered “at random”

- Think of counterparties taking positions “at random” in K independent market factors.
- The number of factors or dimensionality, K, to be determined from empirical data.
- Normalised positions are represented by points on a (K – 1) – sphere:



- Correlation between two positions is the cosine of the angle between them.

Distribution of Correlations

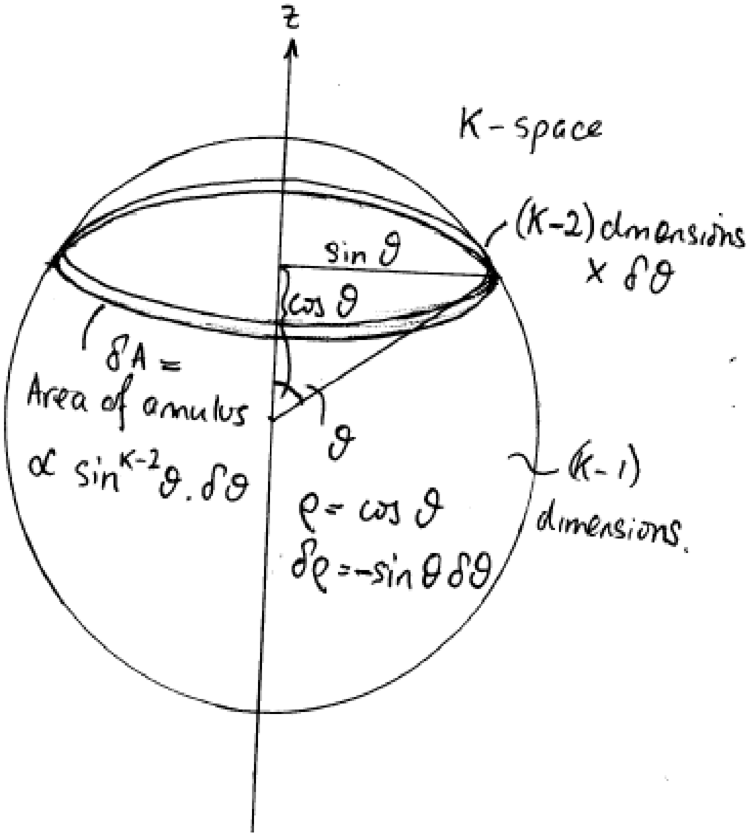
The model implies correlations should be beta distibuted

- The correlation between two points on our sphere is:

$$\rho = \cos\theta$$

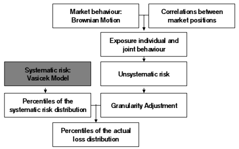
- Where θ is the “angle” between them (the actual angle when K =3).
- Assuming points are scattered at random on the surface, the density is completely determined by the dimension K:

$$g(\rho) = \kappa(K)(1-\rho^2)^{\frac{K-3}{2}}$$



- κ is a normalisation constant depending on K.
- This is just the beta distribution on [-1, +1].
- Reduces to uniform for K = 3.
- Gives the lines which we compared to empirical correlation above.

The Vasicek Model



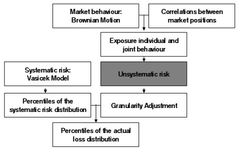
A model that needs no introduction

- This is the best known one factor model.
- Basis of the Basel II IRB risk weights - one factor form of CreditMetrics.
- Popular as has broadly believable realistic properties
- Gives rise to the following well – known form for conditional default probability:

$$P(x) = N(\frac{N^{-1}(p) + \lambda^{1/2} x}{(1-\lambda)^{1/2}})$$

- Where λ is the asset correlation, usually taken to be around 20%.
- p is the average default probability (usually specified in advance).
- Systematic factor *X* is identified with the systematic component of asset return. We take X to be normally distributed, though other choices are of course possible.
- Merton model of individual firm (default occurs when liabilities exceed assets).

Combining Exposure and Default Risk



Passing from exposure to loss statistics

- Formulae are quite analogous to case of loans.

Expected Loss

- We just substitute EPE for notional – EPE is “loan equivalent exposure” in this sense

	Loans	Counterparties	Note
Expected Loss	$\sum_A N_A \times LGD_A \times p_A$	$\sum_A EPE_A \times LGD_A \times p_A$	Just replace EAD with EPE

Systematic risk = Conditional Expected Loss

- We’re more interested in conditional expected loss (will see why later)
- But this is obtained by the same substitution:

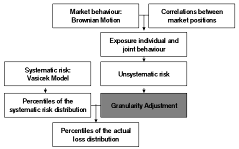
	Loans	Counterparties	Note
Expected loss, *conditional on* X	$\sum_A N_A LGD_A P_A(X)$	$\sum_A EPE_A LGD_A P_A(X)$	Same relation as for unconditional EL

Concentration risk or Conditional Variance

- Everything can’t be the same, as there are additional risks in the counterparty case!
- We have already worked out the additional exposure terms F_A, c_{AB} needed, however.

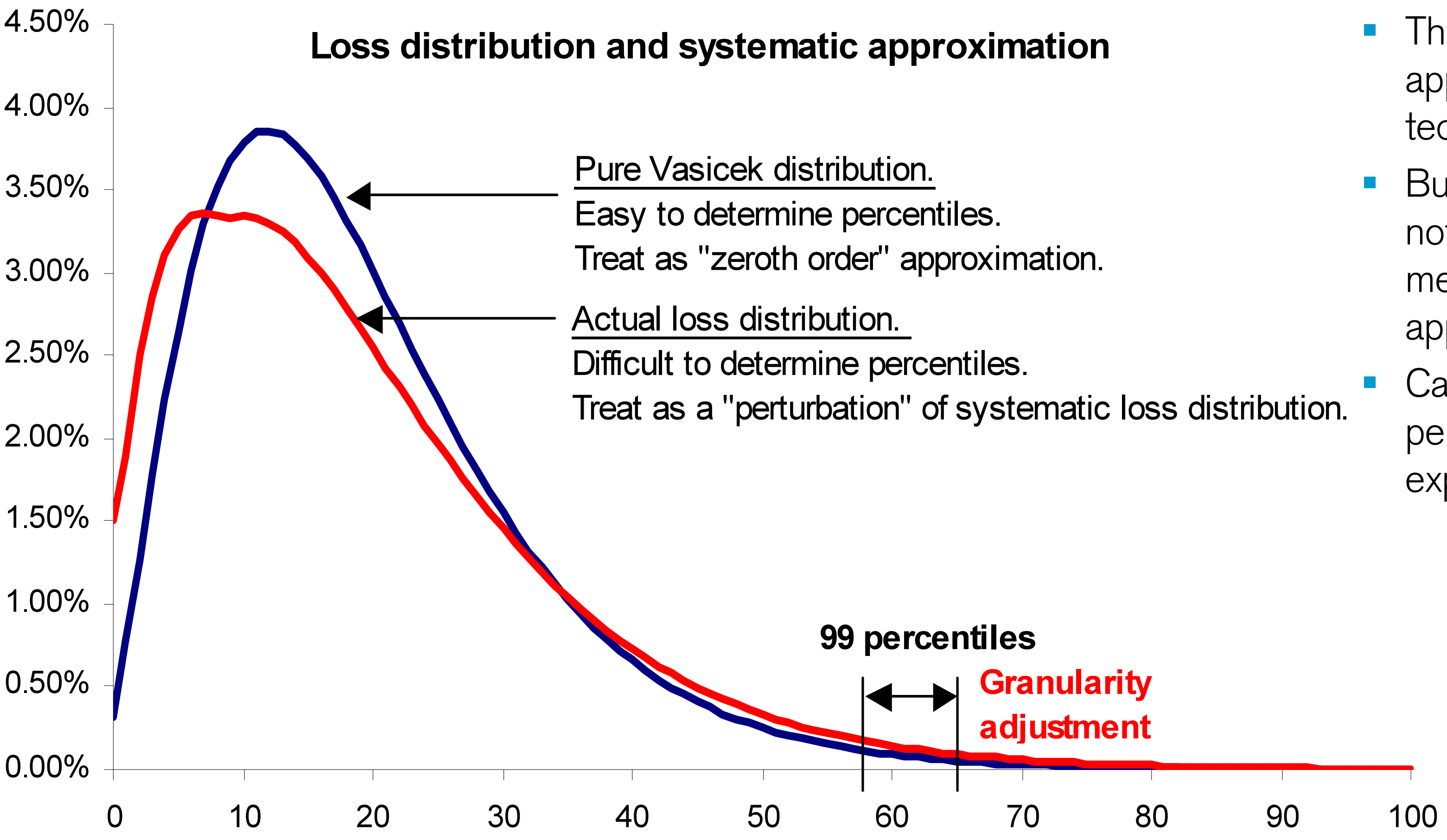
	Loans	Counterparties	Note
Variance, conditional on X	$\sum_A N_A LGD_A P_A (1-P_A)$	$\sum_A F_A^2 P_A - \sum_A E_A^2 P_A^2 + \sum_A c_{AB} P_A P_B$	Not just substitution of EPE - additional risks are present

Granularity Adjustment Concept



Concept

- Actual loss distribution is analytically intractable.
- But sometimes a "nearby" distribution is tractable e.g. the systematic risk distribution.
- Granularity = treat distribution of interest as a "perturbation" of the nearby distribution.
- In effect a Taylor expansion.



Loss distribution and systematic approximation

Pure Vasicek distribution.
Easy to determine percentiles.
Treat as "zeroth order" approximation.

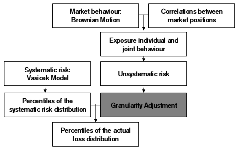
Actual loss distribution.
Difficult to determine percentiles.
Treat as a "perturbation" of systematic loss distribution.

99 percentiles

Granularity adjustment

- This is an approximation technique
- But it is not heuristic – not like e.g. mean/variance approximation
- Can be used for percentiles and for expected shortfall

Granularity Adjustment Mathematics



Market behaviour: Brownian Motion

Correlations between market positions

Exposure individual and joint behaviour

Systematic risk: Vasicek Model

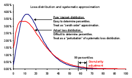
Unsystematic risk

Percentiles of the systematic risk distribution

Granularity Adjustment

Percentiles of the actual loss distribution

Step 1 – Set up

- Y = random variable of interest (in our case ≡ actual losses).
- X = “explanatory variable” (in our case ≡ systematic asset return).
- What does “explanatory variable” mean? Same as in regression analysis, we mean that X explains most of the volatility in Y. We want the conditional variance $\sigma^2(x) = \sigma^2(Y|X = x)$ to be “small”.
- This is actually rather more than the graph  implied. We are not quite saying X and Y have similar densities, but specifying that knowing X says a lot about Y.

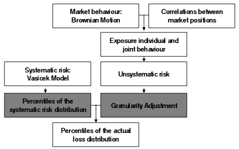
Step 2 - Perturbation

- If $\sigma^2(x)$ were zero, then we would have Y = E(YIX), so if there is some kind of expansion in terms of $\sigma^2(x)$, it had better begin with E(YIX).
- Introduce a “perturbation parameter” ε (will set ε = 1 at the end) and write

$$Y(\varepsilon) = \mathrm{E}(Y|X) + \varepsilon(Y - \mathrm{E}(Y|X)) := \mathrm{E}(Y|X) + \varepsilon U, say$$

- So that Y(1) = Y, the distribution of interest, and Y(0) = E(YIX), the starting point.

Granularity Adjustment Mathematics



Step 3 – Taylor series

- Then we hope that the percentiles Y_q have a Taylor series expansion in powers of ε :

$$Y(\varepsilon)_q = Y(0)_q + \varepsilon \frac{\partial Y_q}{\partial \varepsilon}\bigg|_{\varepsilon=0} + \frac{1}{2}\varepsilon^2 \frac{\partial^2 Y_q}{\partial \varepsilon^2}\bigg|_{\varepsilon=0} + \ldots$$

- Setting $\varepsilon = 1$ gives and substituting for $Y(0)^1$ gives:

$$Y_q = E(Y \big| X = X_q) + \frac{\partial Y_q}{\partial \varepsilon}\bigg|_{\varepsilon=0} + \frac{1}{2}\frac{\partial^2 Y_q}{\partial \varepsilon^2}\bigg|_{\varepsilon=0} + \ldots$$

- Note that we have already calculated the leading term

$$E(Y \big| X = X_q) = \sum_A EPE_A \times LGD_A \times P_A(X_q)$$

- In Basel speak this term is obtained by applying the IRB risk weights to EPE and summing - indeed ISDA's original proposal was to do just that.
- To get the higher terms, need calculable expressions for the **"Derivatives of VaR"**

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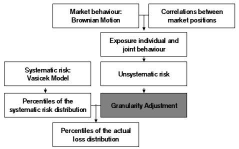
[1] We are cheating slightly here – to make the substitution $E(Y|X_q) = E(Y|X)_q$ we need E(YIX) to be a monotone increasing function of X. However this is satisfied. This point is dealt with by e.g. Gordy (2003).

Granularity Adjustment Mathematics



Step 4 – derivatives of VaR

- Second derivative appears due to Gourieroux, Laurent and Scaillet (2001). Several authors for the first derivative from about 1999.
- Key point: the first derivative vanishes identically in our case:

$$\mathrm{E}(U|X=X_q)=\mathrm{E}(Y|X=X_q)-\mathrm{E}(\mathrm{E}(Y|X)|X=X_q))=0$$

DERIVATIVES OF VAR

First derivative

$$\left.\frac{\partial Y_q}{\partial \varepsilon}\right|_{\varepsilon=0}=\mathbb{E}(U|X=X_q)=\mu_1(x)$$

This is identically zero, since E(U|X = x) = 0 for all x. The vanishing of this derivative is the essence of the granularity method.

Second derivative = Granularity Adjustment

$$\left.\frac{\partial^2 Y_q}{\partial \varepsilon^2}\right|_{\varepsilon=0}=-\frac{1}{f_X}\left.\frac{d(f_X\sigma^2(x))}{dx}\right|_{x=X_q}$$

This gives the first non zero term which is exactly the "granularity adjustment".

***n* th derivative - closely related to the "Cornish Fisher expansion", but we won't need this !**

$$\left.\frac{d^m Y_q}{d\varepsilon^m}\right|_{\varepsilon=0}=(-1)^m\sum_{\substack{p\prec m\\ q\prec s\leq|p|-1}}\frac{\alpha_p\alpha_{\hat{q}}(|p|+|q|-1)!}{(s+|q|)!(|p|-1-s)!}(-f)^{-|p|-|q|}\left(\prod_{j=1}^{s}\left(\frac{d^j f_X}{dx^j}\right)^{e_{qj}}\right)\frac{d^{|p|-1-s}}{dx^{|p|-1-s}}\left(\prod_{i=1}^{m}\left(\frac{d^{i-1}(\mu_i f_X)}{dx^{i-1}}\right)^{e_{pi}}\right)$$

means p is a partition of m

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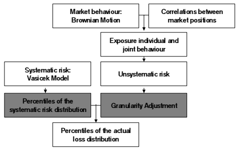
Summary



Calculating counterparty risk

- It is now possible to calculate loss percentiles using this method.
- The granularity method gives the required loss percentile, approximately, as:

$$Y_q = \mathrm{E}(Y|X = X_q) - \frac{1}{2f_X}\frac{d(f_X\sigma^2(x))}{dx}\bigg|_{x=X_q}$$

First term = systematic risk

- The first term is the "systematic risk", and is given as a sum over counterparties.
- It is what you get by treating each counterparty portfolio as a loan, with "loan equivalent exposure" equal to EPE:

$$\mathrm{E}(Y|X = X_q) = \sum_A EPE_A \times LGD_A \times P_A(X_q)$$

Second term = concentration risk(s)

- The second term depends on the conditional variance which we worked out above.
- This encodes information about the volatilities and correlations of market positions in the portfolio:

$$\sigma^2(x) = LGD_A^2\left(\sum_A F_A^2 P_A - \sum_A E_A^2 P_A^2 + \sum_A c_{AB} P_A P_B\right)$$

Systematic LGD risk

Systematic LGD risk

- What if LGD is positively correlated to default rates?
- Then, capital requirements should reflect higher LGDs. In modelling terms, should couple LGD to the systematic factor X, as well as coupling PD to X.
- This is a "hot topic" – see e.g. Frye , "A false sense of security", RISK, August 2003.
- The evidence comes from loans, but may also be true for counterparty risk.

The first order effect –use "stressed LGD"

- The "first order" correction to capital to account for this coupling is, not surprisingly, just to use LGD conditional on X = Xq:
- Capital requirement = Exposure × $\mathrm{LGD}(X_{99.9\%}) \times \mathrm{PD}\ (X_{99.9\%})$

Any other effects?

- For counterparty risk, is there a further effect?
- Capital requirement = $a(X_{99.9\%}) \times \mathrm{EPE} \times \mathrm{LGD}(X_{99.9\%}) \times \mathrm{PD}\ (X_{99.9\%})$?

Systematic LGD risk

The answer is no, not really,

- We modified our analysis to include systematic LGD with linear dependence on X:

$$LGD(x) = LGD(X_q)(1 + \psi_A(x - X_q))$$

- The analysis shows that α actually decreases with this coupling (though the effect is small in most cases).