Linear interpolation
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Interpolation is a means of determining the value of an unknown data point based on the values of known surrounding data points. Linear interpolation is a simple method of interpolation that assumes a straight line (linear) relationship between the known points; it essentially means averaging the two rates over the interpolation period.

Financial market participants often use interpolation methods, including linear interpolation, to determine the value of missing interest rates on a yield curve. For example, assume that a bank wants to calculate the present value of a payment it will receive 1½ months from today; the calculation will require an interest rate appropriate to the time to receipt of the cash flow. Because there is no 1½-month rate, but only one-month and two-month rates, it will be necessary to interpolate the required rate.

Interpolation also might be necessary for calculating a payment for a reset date falling between standard quoted periods, for example, at the beginning or end of a transaction. The 2000 ISDA Definitions define linear interpolation as follows (Section 8.3):

...the Relevant Rate for the Reset Date in respect of that Calculation Period or any Compounding Period included in that Calculation Period shall be determined through the use of straight-line interpolation by reference to two rates based on the relevant Floating Rate Option, one of which shall be determined as if the Designated Maturity were the period of time for which rates are available next shorter than the length of the Calculation Period or Compounding Period (or any alternative Designated Maturity agreed to by the parties) and the other of which shall be determined as if the Designated Maturity were the period of time for which rates are available next longer than the length of the Calculation Period or Compounding Period (or any alternative Designated Maturity agreed to by the parties).

Put somewhat differently, if the unknown rate lies between two known rates, linear interpolation assumes the unknown rate lies on a straight line between the two. The following illustrates how this works.

Today’s date is December 5, 2005, and a bank needs to determine a floating rate payment to be made on January 19, 2006, which is 45 days from today (t_r). Rate source is BBA Libor. Normally one uses today’s Libor rate to determine the payment, but there is no 45-day Libor rate available. The closest one can get is one-month Libor (R_1),
equal to 4.3313\% and settling 31 days from today (t_1); and two-month Libor (R_2), equal to 4.3944\% and settling 63 days from today (t_2).

Figure 1 shows the relationship between the two rates and days to maturity. Linear interpolation assumes that the unknown 45-day rate (R_n) lies on the line (AC) between the two known rates. Because AC is a straight line, that is, linear, the slope of the line (AB) connecting R_1 and R_n is the same as the slope of line AC. Using the "rise over run" formula for the slope of the line, we determine R_n as follows:

\[
R_n = R_1 + \frac{R_2 - R_1}{t_2 - t_1} \times (t_n - t_1)
\]

\[
= 4.3313\% + \frac{4.3944\% - 4.3313\%}{63 - 31} \times (45 - 31)
\]

\[
= 4.3313\% + 0.00197\% \times (14) = 4.3589\%
\]

The following is the derivation of the general version of the above equation. Given two known rates and their maturities:

- \(R_1\) = Known rate with shorter maturity
- \(t_1\) = Number of days to maturity of \(R_1\)
- \(R_2\) = Known rate with longer maturity
- \(t_2\) = Number of days to maturity of \(R_2\)
- \(R_n\) = Unknown rate between \(R_1\) and \(R_2\)
- \(t_n\) = Number of days to cash flow at time \(n\)

Find unknown rate \(R_n\) with known maturity \(t_n\) \((t_1 < t_n < t_2)\) based on the assumption that the slope of line AB is equal to the slope of line AC as follows.

\[
\frac{R_n - R_1}{t_n - t_1} = \frac{R_2 - R_1}{t_2 - t_1}
\]

\[
\Rightarrow R_n - R_1 = \frac{R_2 - R_1}{t_2 - t_1} \times (t_n - t_1)
\]

\[
\Rightarrow R_n = R_1 + \frac{R_2 - R_1}{t_2 - t_1} \times (t_n - t_1)
\]

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Another commonly used—and mathematically equivalent—version of the linear interpolation formula is the following:

\[ R_n = \frac{(R_1 \times (t_2 - t_n)) + (R_2 \times (t_n - t_1))}{t_2 - t_1} \]

In the preceding example, the solution would be as follows:

\[ R_n = \frac{(4.3313\% \times (64 - 45)) + (4.3944\% \times (45 - 31))}{63 - 31} = 4.3589\% \]

**Determining days to maturity**

In the above example, it is necessary to provide number of days to maturity for the two unknown rates. The market practice for determining the number of days to maturity for each rate is to follow the relevant conventions for the rate option used. The preceding example involves Libor rates of one month and higher, so the relevant convention is to determine days to maturity according to the Modified Following business day convention for London (and Following for periods of less than one month). Because February 5, 2006, falls on a Sunday, February 6 is the next good London business day and becomes the settlement date.

With the exception of ISDAFIX, most rate options are administered by organizations other than ISDA. Libor, for example, is administered by the British Bankers Association, while Euribor is administered by the European Banking Federation. Click here to learn more about maturity dates and business day conventions for Libor; click here to obtain similar information for Euribor.