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Mrs Danièle Nouy
Chairman
Models Task Force
Basel Committee of Banking Supervision
Bank for International Settlements
Central Bahnplatz 2
CH-4051 Basel
Switzerland

Dear Mrs Nouy,

RE : Comments on the granularity adjustment

In its response to the Basel Committee's second consultation paper on the new capital adequacy framework, ISDA noted the need for additional work to be performed on the shaping of the granularity adjustment.

We continue to believe that there is conceptual merit in introducing a granularity adjustment, although the MIS implications of it should not be underestimated. For some institutions, producing the aggregate lending exposure to each non-retail counterparty will induce additional and substantial costs, that should be borne in mind by the regulators when implementing the approach.

Our first comment, arising from observations of banks completing the Quantitative Impact Study ("QIS"), is that the degree of accuracy with which the granularity adjustment will be able to be quoted is low. This is because of the way the adjustment is calculated. The adjustment ("GA") is given by the following formula (New Accord, III, ¶514-5):

$$GA = TNRE \times GSF / n^* - 4\% \times RWA$$

This shows the adjustment to be the net of two very large numbers, itself typically quite small. Treating risk weighted assets RWA as certain, then a small proportional uncertainty in GSF or n^* will be leveraged into a very large proportional uncertainty in the adjustment itself. For example, it may not be determinable whether the adjustment is positive or negative, and regulators will have to accept statements like "between - $100m and + $100m" for the GA which would not normally be acceptable in terms of accuracy.

It should further be noted that the adjustment does not, as currently presented, account for country/industry diversification, which significantly reduces its meaningfulness from an industry perspective.

Notwithstanding the points above, ISDA would like to suggest a number of changes to the Committee's approach to setting a granularity adjustment, with a view to rendering the calculation more compatible with the model underpinning the IRB charge, as well as avoiding a number of negative side-effects, particularly with respect to the inclusion of *LGD* and the volatility of *LGD* (*VLGD*) in the formulas.

- **Correctness of the functions proposed for GSF and n* (New Accord, III, ¶514-5)**

Reminder

The granularity adjustment is an additional capital requirement to cope with unsystematic credit risk arising from the lending portfolio. It is by means of this adjustment that the Basel Committee intends to require additional capital for portfolios which contain very large or varied exposure sizes, relative to those which are more "granular" or contain large numbers of smaller exposures.

For a homogeneous portfolio of *N* obligors, the work referenced in the New Accord shows numerically that $GA = L - L_\infty = \beta / N$ where *N* is the number of exposures in the portfolio and β is the "slope" (New Accord, V, ¶448-9). This is extended to a non-homogeneous portfolio by replacing *N* with the "effective number of obligors" n^*, where $n^* = N$ for a homogeneous portfolio. Thus the required adjustment, expressed as % of total exposure, is specified as the ratio of the slope parameter β and the effective number of obligors.

$$GA = L - L_\infty = \beta / n^*$$

The final formula for the granularity adjustment is developed from this formula by specifying the slope β. The New Accord gives the value of the β as

$$\beta = (0.4 + 1.2LGD_{AG})(0.76 + 1.1PD_{AG} / F_{AG}) \text{ (New Accord, V, ¶456)}$$

After scaling by $18.75 = 1.5 \times 1 / 0.08$, being the product of the current 150% scaling factor and the adjustment from capital to risk weighted assets, the formula for β becomes the formula for the granularity scaling factor (New Accord, V, ¶457):

$$GSF = (0.6 + 1.8LGD_{AG})(9.5 + 13.75PD_{AG} / F_{AG})$$

The adjustment is then

$$GA = TNRE \times GSF / n^* - 4\% \times RWA \quad \text{(New Accord, V, ¶433)}$$

The last term rebates 4% of risk weighted assets against the adjustment. This cancels the charge of 4% levied for granularity within the base risk weights (New Accord, V, ¶457).

Correctness of the function proposed

In order to derive the granularity adjustment, the Basel Committee uses a modelling framework (Creditrisk+) distinct from that underpinning the IRB function (New Accord, V, ¶444). ISDA believes that use of the Vasicek framework throughout both the IRB function and the granularity adjustment is to be preferred on the grounds of consistency. Further we believe this approach produces the following quite different coefficients in the granularity adjustment :

$$\beta_{Vasicek} = (0.4 + 1.2LGD_{AG})(0.29 + 4.29PD_{AG} / F_{AG})$$

Our view is based on a theoretical derivation verified by numerical work (see Annex). With the current calibration of the risk weights, the same multiplier of $18.75 = 1.5 \times 1 / 0.08$ would apply to our formula giving:

$$GSF = (0.6 + 1.8LGD_{AG})(3.6 + 53.6PD_{AG} / F_{AG})$$

This formula produces different results to the currently proposed formula, although it should be noted that the difference is often fairly small. This is because the fraction *PD/F* is not very sensitive to the actual portfolio. The two formulae coincide for a default probability of roughly 2% and are within roughly 10% of one another for PD_{AG} in the range 1.1% to 3.5%. The Vasicek based formula gives a higher granularity adjustment for default probabilities in excess of 2%, and otherwise lower, all else being equal.

Note on non homogeneous portfolios
The analysis (see Annex) further indicates that the "real" formulae are not in the form β / n^* when the portfolio is not homogeneous although they reduce to this case for a homogeneous portfolio. However the "real" form in the non-homogeneous Vasicek case is complex and the error introduced may well not be big enough to justify using the full formula.

- **Presentation of the calculation, in particular use of PD bands *b* (New Accord, III, ¶507) and embedded assumption about volatility of LGD (New Accord, V, ¶447)**

Use of PD bands
The "bands" *b* used in the calculation (New Accord, V, ¶449) should properly contain only assets with a common *LGD* as well as *PD*. Provided all the assets in each band have equal *LGD* and *PD*, then the calculation does not depend in any other way on the precise banding used, but if exposures having a different *LGD* and/or *PD* are put into one band, then the final adjustment depends on the band structure, which is not appropriate. ISDA suggest that the simplest approach is to avoid phrasing the calculation in terms of bands, which are not in any case necessary to the calculation, and instead to simply refer directly to counterparty level exposures.

Particular note on secured exposures (e.g. secured financing)
The New Accord exacerbates this problem by requiring the *LGD*, rather than the *EAD*, to be altered to reflect the presence of collateral (New Accord, III, ¶201). For example, under the IRB foundation approach with standard 50% *LGD*, the *modified LGD** for a repo transaction under ¶201 might be, say 5%. It could be argued that the *EAD*, rather than *LGD* should be modified, but for the calculation of the base capital requirement only the product of *EAD* and *LGD* is relevant, so the choice of which of *EAD* and *LGD* to alter for collateral has no impact. However when the granularity adjustment calculation is performed, an impact does arise.

First, the fact that secured transactions have modified *LGD*'s significantly different from the standard 50% assumption naturally exacerbates the inaccuracy generated by averaging within band. On the other hand, if a bank tries to improve the accuracy of the calculation by using separate bands *b* for secured transactions with low *LGD*'s, and thus creates separate bands for these (the New Accord does not prohibit this), then the granularity adjustment can be materially overstated due to an interaction with *VLGD* assumption, as explained below.

Assumption about volatility of LGD
The granularity adjustment includes adjustment for unsystematic recovery rate volatility, for which the following expression is assumed for volatility as a % of exposure (New Accord, V, ¶447):

$$VLGD = 0.5\sqrt{LGD(1-LGD)}$$

For *LGD* = 50%, the standard assumption for unsecured exposures it gives a reasonable 25% volatility. However for secured exposures the picture is different due to the requirement under ¶201 to modify *LGD* to reflect security. This is most easily seen in terms of proportional volatility, i.e. *VLGD* as a % of *LGD*, which is given by

$$\%VLGD = 0.5\sqrt{(1-LGD)/LGD}$$

For small *LGD*, the proportional volatility becomes very large. Thus when *LGD* = 5%, *VLGD* = 11% and proportional *VLGD* = 220%, which is more than 4 times larger than at 50% *LGD*. In effect, this is an additional haircut for collateral, whose volatility will have been taken account of in full via the initial haircut under the comprehensive approach. As we mentioned above this effect will only be felt fully when secured transactions with low *LGD*'s are bucketed *separately* from other transactions. If not, protection will be afforded by the averaging out of *LGD*.

ISDA would suggest retaining a simpler approach to *LGD* volatility in the form of the following proportional relationship:

$VLGD = 0.5LGD$, for LGD<=50%, and

$VLGD = 0.5 - 0.5\ LGD$, for LGD>50%

For LGD = 50%, this would as before produce VLGD = 25%. However, collateralised transactions would not be charged extremely high volatilities as suggested above.

ISDA hopes that the Basel Committee, and particularly the Models Task Force, will find the comments above useful. As always, we would be happy to discuss further, should this be deemed appropriate,

Yours sincerely,

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The New Accord's Granularity Adjustment[1]

By Tom Wilde, CSFB

Introduction[2]

In a recent article in RISK magazine (Wilde, 2001) I attempted to explain the IRB approach to assessing capital for credit risk from the New Accord (Basel, 2001). I showed that the granularity adjustment seemed reasonable on the heuristic basis that it was similar to the result of scaling based on standard deviations.

The numerical analysis behind the granularity adjustment is explained in Gordy, 2001. There is a relatively weak link in the application of this analysis to the New Accord[3]: In the calculation of the adjustment the Vasicek model which was used to calibrate the base risk weights is discarded in favour of an enhanced form of CREDITRISK+. This is justified (Basel, 2001, paragraph 444) only by the statement that the two models have tails which align closely. However, the cautious investigator might be concerned that the adjustment depends on subtle tail shape aspects which are not similar, and in fact this appears to be the case as I will attempt to demonstrate.

In this sequel article I reconsider the granularity adjustment from a more critical point of view: bearing in mind that the adjustment seems roughly right, we now ask: How accurate is it, and what adjustment would have been obtained using the Vasicek model throughout?

The granularity adjustment

The granularity adjustment is given in the New Accord, by the following formula

$$\beta_{Actual} = (0.4 + 1.2LGD)(0.76 + 1.1\frac{PD}{F}) \qquad (1)$$

See Basel 2001, paragraph 456. The precise meaning of β is introduced at paragraph 442. In this equation, PD is the default probability and $F = N(1.118N^{-1}(PD) + 1.288) - PD$ is the systematic risk sensitivity (Basel, 2001, paragraph 431)[4]. Formula (1) is then scaled by a factor of $1.5 \times 1/0.08 = 18.75$, where the factor of one and a half is the same arbitrary scaling factor currently applied to the base risk weights, to give the granularity scaling factor (*"GSF"*) actually used for the calculation (Basel, 2001, paragraph 457). We will work directly with equation (1).

Summary of main conclusions

The main results and conclusions are as follows:

Theoretical granularity adjustment

I present a theoretical general formula for the granularity adjustment. Computing this formula for the CREDITRISK+ model used in the New Accord gives the following "theoretical granularity adjustment"

$$\beta_{Theoretical}^{CreditRisk+} = (0.4 + 1.2LGD)(0.762 + 1.075\frac{PD}{F}) \qquad (2)$$

in striking agreement with the New Accord's numerically derived granularity adjustment (1).

Theoretical granularity adjustment for the Vasicek distribution

Having verified the formula in the case of the CREDITRISK+ model we naturally ask what result it gives for the Vasicek distribution. By a similar calculation to equation (2) we get:

$$\beta_{Theoretical}^{Vasicek} = (0.4 + 1.2LGD)(0.29 + 4.29\frac{PD}{F}) \qquad (3)$$

I concluded before that based on standard deviations the granularity adjustment seems reasonable

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[1] This Annex was produced by Tom Wilde, Credit Suisse First Boston. A version of it will soon be published in Risk Magazine.

[2] I am very grateful to Michael Gordy for corrections and improvements on earlier versions of this article, as well as for stimulating this work through Gordy, 2001 and the relevant parts of the New Accord

[3] It is to be stressed that this weakness is in the application of the methods of Gordy, 2001, not in the methods themselves, some of which the results of this paper serve to verify.

[4] In the application in the New Accord *PD, LGD* and *F* are actually averages (in the theoretical work they are not). The averaging process brings issues of its own which are remarked on in the Section headed "Note

(Wilde, 2001) That conclusion is consistent with these results. Surprisingly, equations (1) or (3) are quite similar for most portfolios because the fraction *PD/F* is quite insensitive to *PD*, so the coefficients in (1)-(3) have only a mild effect on the results. (1) and (3) coincide for $PD = 2\%$ approximately and are within roughly 10% of one another for *PD* in the range 1.1% to 3.5%. Formula (3) gives a higher adjustment for default rates above 2%, otherwise a lower one.

Formula (3) has a claim to be called more correct than formulae (1) or (2) because its derivation does not involve a switch of model, so that unlike the New Accord's formula (1), it is fully consistent with the rest of the IRB approach. Unfortunately, that does not really mean formula (3) is "right": One could argue that the IRB approach could have been built on a CREDITRISK+ framework, in which case formula (1) would have been the correct choice instead. This difference between the models cannot be made to go away since all available parameters have been used to calibrate the models at their 99.5% percentiles. Thus, we have shown that the adjustment is intrinsically model dependent. This conclusion is, of course mitigated by the fact that the results of formulae (1) - (3) are comparable over a wide range of default probabilities.

Numerical work

At the end of this article, numerical work is presented to check that (3) actually does coincide with the slope of the adjustment when the Vasicek distribution is used. This is an appropriate check as the theoretical result (equation (4) below) is a "first order approximation". Agreement with numerical results is good. The agreement between (1) and (2) can be viewed as a numerical check on the formula in the CREDITRISK$^+$ case.

The Theoretical Granularity Adjustment Formula

The New Accord is built on the idea of a one factor credit risk model, where the base risk weights measure the systematic risk present. To state the formula providing a theoretical value for the granularity adjustment, we need to set up some notation. I have tried to be consistent with my earlier article. Thus suppose that Y is a loss distribution with X as the systematic factor, and with conditional mean and variance given by

$$\mu(x) = \mu(Y|X = x) \text{ and } \sigma^2(x) = \sigma^2(Y|X = x)$$

The distribution of $\mu(X)$ is the systematic loss distribution. We use (upper case) P to denote the default probability of an obligor as a function of X and (lower case) p to denote its average value, i.e. the unconditional default probability of A. Let the probability density function of X be $f_X(x)$. Let X_q be the percentile of X corresponding to confidence level $1 - q$ (e.g. in the case of the gamma distribution used in the New Accord, $q = 0.5\%$ i.e. $1 - q = 99.5\%$, and $X_q = 12.007$ is the corresponding value of X). The corresponding systematic percentile of Y is $L = \mu(X_q)$. Let $L + \Delta L$ be the actual percentile Y_q of Y. Then ΔL is the "granularity adjustment". Then the following formula gives a theoretical first order approximation to the granularity adjustment, with the understanding that the right hand side is to be evaluated at $x = X_q$.

$$\Delta L \cong \frac{-1}{2f_X}\frac{d}{dx}\left(\frac{f_X\sigma^2}{d\mu/dx}\right) \tag{4}$$

To be more precise, we are saying that equation (4) gives the granularity adjustment to "first order in the unsystematic variance". Unfortunately it is tricky to formulate exactly what this means because for any actual portfolio the adjustment is not a function of the unsystematic variance only, but clearly depends on the detailed composition of the portfolio. One way of avoiding this difficulty is to use the systematic risk process described in Wilde, 2001[5]. We start with a portfolio $\Pi = \Pi_1$ with conditional mean and variance $\mu(x)$ and $\sigma^2(x)$ and construct a sequence of portfolios Π_m by replacing each obligor $A \in \Pi$ with m new obligors $A_1, \ldots, A_m$ each having exposure E_A/m but the same default probability and proportional recovery rate volatility as A. As m tends to infinity the loss distributions of Π_m tend to the systematic loss distribution $\mu(X)$. and we can notionally associate this with a "systematic portfolio" Π_∞. Then for every value of X the conditional unsystematic variance of Π_m satisfies $\sigma_m^2(x) = \sigma^2(x)/m$, and moreover since we

[5] ... systematic limit process is discussed in Gordy, 2001

specified the starting portfolio $\Pi = \Pi_1$ everything is just a function of m. This allows us to write equation (4) in the following more precise form, where $L^{(m)}$ is the percentile of Π_m and $L = L^{(\infty)}$ is the systematic percentile:

$$L^{(m)} - L^{(\infty)} = \frac{-1}{2mf_X}\frac{d}{dx}\left(\frac{f_X\sigma^2}{d\mu/dx}\right)\Bigg|_{x=L^{(\infty)}} + O(1/m^2) \tag{5}$$

If $\Pi = \Pi_1$ is a single obligor with unit exposure then Π_m is just a homogeneous portfolio with m obligors each having exposure $1/m$, i.e. with total exposure 1 unit. From the definition of the slope β which is the subject of equation (1) (see Basel, 2001, paragraph 443), we see that (5) is exactly equivalent to the following statement:

$$\beta = \frac{-1}{2f_X}\frac{d}{dx}\left(\frac{f_X\sigma^2}{d\mu/dx}\right)\Bigg|_{x=L^{(\infty)}} \tag{6}$$

In other words, we are giving a formula for the slope of the lines on Chart 7 of the New Accord[6] at the origin[7]. Here μ and σ^2 are for Π_1, which is a single obligor with unit exposure. This sounds odd but note that the term in brackets doesn't depend on the number m of obligors since μ and σ^2 scale by m, so their ratio is the same for all Π_m in the sequence.

Deriving Equation (4)

First we show that we only need a special case. Intuitively, the systematic variable X is a dummy variable, so the precise form of the function $\mu(X)$ should not be relevant. So as we show in a moment, while $\mu(X)$ can be any monotonic increasing function of the systematic variable X, we may actually assume $\mu(X) = X$. In this case, the formula takes the form

$$\Delta L \cong \frac{-1}{2f_\mu}\frac{d(f_\mu\sigma^2)}{d\mu} \tag{7}$$

This is sufficient because it implies (4) as follows. For any monotonic increasing $\mu = \mu(X)$ we have for the probability densities $f_\mu = f_X dx/d\mu$ and substituting this into (7) we get

$$\Delta L \cong \frac{-1}{2f_X dx/d\mu}\frac{d(f_X\sigma^2 dx/d\mu)}{d\mu} = \frac{-1}{2f_X}\frac{d}{dx}\left(\frac{f_X\sigma^2}{d\mu/dx}\right)$$

which is (4). As a further simplification, instead of looking at percentiles we look instead at the amount of "extra probability" at the percentile. In fact, let $q + \Delta q$ be the probability that the actual loss will exceed L, the $1 - q$ systematic risk percentile (By definition, the systematic loss variable has probability q of exceeding L). We will actually justify the following result

$$\Delta q \cong -1/2\frac{d(f_\mu\sigma^2)}{d\mu} \tag{8}$$

Intuitively, ΔL is the amount by which we have to shift the percentile to the right in order to compensate for the extra probability Δq, so we expect $\Delta q = f(L)\Delta L$ to first order in ΔL from which (4) clearly follows from (8). This isn't quite right because f is the density of X, not Y. A more careful analysis shows that the right equation is $f(L)\Delta L = \Delta q(L + \Delta L)$ where the right hand side means Δq evaluated at $L + \Delta L$. To first order this is $\Delta L \times (f(L) - d\Delta q/d\mu) = \Delta q(L)$, instead of $\Delta q = f(L)\Delta L$. Reflecting this properly would therefore introduce an adjustment $(1 - f(L)^{-1} d\Delta q/d\mu)^{-1}$ into equations (4) – (6). But differentiating equation (8) shows that $d\Delta q/d\mu$, like Δq, is $O(1/m)$ where m is as in the limit process above. Hence the adjustment factor is $1 + O(1/m)$ and so the original equation $\Delta q = f(L)\Delta L$ is good enough to first order[8]. Now we focus on equation (8). To give some

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[6] This is a rigorous statement and I believe a rigorous proof exists under mild conditions (essentially existence of the right hand side), but we are going to give just a convincing mathematical argument below.

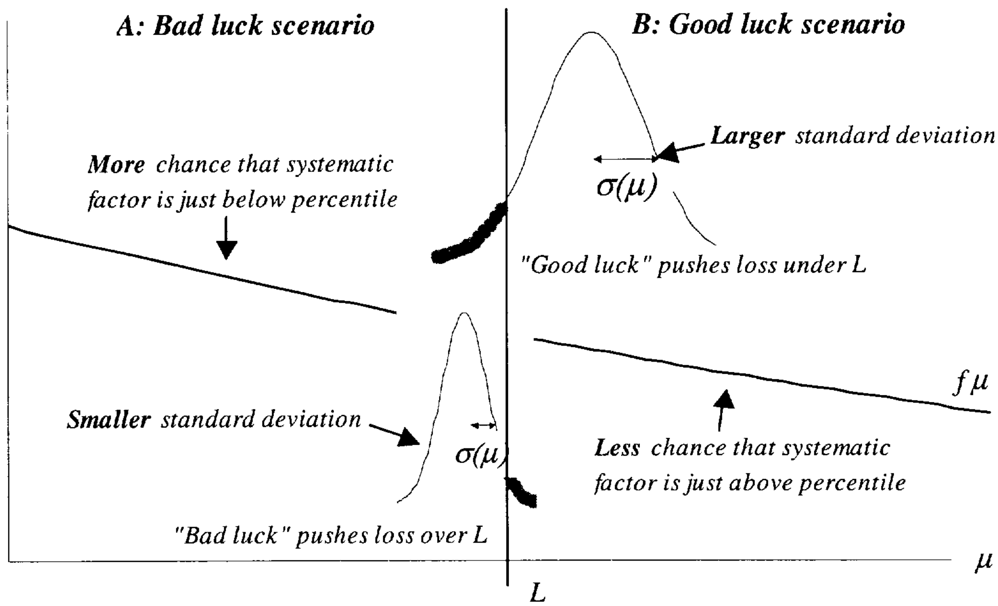
[7] Except that our formula is valid whether the starting portfolio is homogeneous or not.

[8] Or in summary, the our original intuition that the density functions of X and Y are similar was correct.

motivation rewrite equation (8) as

$$\Delta q \cong -1/2(\sigma^2 \frac{df_\mu}{d\mu} + f_\mu \frac{d\sigma^2}{d\mu}) \tag{9}$$

There are two terms. Since we are looking at the tail of the density function f_μ, one would expect the first term in brackets to be negative (the density slopes downwards in the tail). On the other hand from the usual formula for variance we know it is normally increasing as a function of the default probabilities, meaning that the second summand should be increasing. The two summands represent offsetting effects which can easily be described with the aid of the diagram below.



To experience a loss over *L*, if there is not too much unsystematic risk present, the systematic factor "must" be close to *L*. Poor economic conditions are all but required to get a 99.5% loss; in all but the smallest portfolios the chance of such a loss by sheer bad luck is negligible. If the systematic factor is *slightly* below *L*, losses might be "pushed above" *L* by a small amount of bad luck (i.e. accidental defaults and poor recovery rates); conversely there is a chance of being saved by good luck when the factor is slightly above *L*. These offset as follows (see diagram)

- there is slightly more chance of the factor being just below than just above *L*, so the bad luck scenario *A* is more likely. The imbalance contributes a positive term $-1/2\sigma^2 df_\mu / d\mu$
- on the other hand the good luck scenario *B* is more powerful, because volatility (including the benefit of a lower than expected loss outcome) is greater when default rates are higher. This contributes the negative offsetting term $-1/2 f_\mu d\sigma^2 / d\mu$

Hoping that this analysis has provided enough motivation we now examine a mathematical demonstration of equation (8). We have been able to assume $\mu(x) = x$: In particular *L* is the $(1-q)$ - percentile of *X*. We write $x = L + t$ to centre the integration around *L*, and have quite generally

$$q + \Delta q = \int_{t=-\infty}^{\infty} P(Loss \geq L | x = L + t) P(x = L + t) dt$$

where *P* in the integrand stands for probability. Since *q* is the probability that $x > L$, we also have

$$\Delta q = \int_{t=-\infty}^{0} P(Loss \geq L | \mu = L + t) P(x = L + t) dt + \int_{t=0}^{\infty} (P(Loss \geq L | x = L + t) - 1) P(x = L + t) dt$$

We now make three approximations, all justified by closeness to *L*. First consider the limit process described above before equation (5). Passing from $\Pi = \Pi_1$ to Π_m replaces each obligor with *m* independent obligors, having *m* identical independent loss outcomes. The conditional distribution of losses is therefore asymptotically normal for every value of μ and so it is plausible to approximate the conditional probabilities in the integrands by cumulative normal densities[9]. Since the conditional average $\mu(x) = x$ we have, using the symmetry of the normal distribution:

$$P(Loss \geq L \mid x = L + t) = 1 - N(-t/\sigma(L+t)) = N(t/\sigma(L+t))$$

where N is the cumulative normal function. Next, if the unsystematic risk is small (i.e. m is large in equation (5)) then the problem is "local" in that the systematic variable μ has to be close to L to give any significant probability of crossing over L. We can therefore safely make linear approximations to the integrands which are valid only near L, provided those approximations do not "blow up" for values of x far from L. In particular, we can think of the density f as linear (as it looks on the diagram above), and we can think of the standard deviation $\sigma(L+t)$ as linear. Putting all these approximations together we can write

$$\Delta q = \int_{-\infty}^{\infty} (f(L) + tf'(L)) G(\frac{t}{\sigma(L+t)}) dt$$

where we have introduced G defined by

$$G(y) = \begin{cases} N(y) - 1 & y > 0 \\ N(y) & y \leq 0 \end{cases}$$

Because of the last approximation

$$\frac{t}{\sigma(L+t)} = \frac{t}{\sigma(L)}(1 - \frac{t}{\sigma(L)} \frac{d\sigma}{dt}(t=0))$$

In what follows $\sigma'(L)$ refers to the derivative $d\sigma/dt(0) = d\sigma/d\mu(L)$. Hence,

$$\Delta q = \int_{-\infty}^{\infty} (f(L) + tf'(L)) G(\frac{t}{\sigma(L)}(1 - \frac{t}{\sigma(L)}\sigma'(L))) dt$$

Next, by splitting the integral into positive and negative x and substituting $-x$ for x in the positive half we get the equivalent formula $\Delta q = \Delta q_1 - \Delta q_2$, where

$$\Delta q_{1,2} = \int_{-\infty}^{0} (f(L) \pm tf'(L)) N(\frac{t}{\sigma(L)}(1 \mp \frac{t}{\sigma(L)}\sigma'(L))) dt$$

All we have to do now is evaluate this integral. As a further first order approximation write[10]

$$N(\frac{t}{\sigma(L)}(1 \mp \frac{t}{\sigma(L)}\sigma'(L))) = N(\frac{t}{\sigma(L)}) \mp \frac{t^2}{\sigma^2(L)}\sigma'(L) n(\frac{t}{\sigma(L)})$$

where $n(x) = dN/dx$ is the normal density function. We obtain

$$\Delta q_{1,2} = \int_{-\infty}^{0} (f(L) \pm tf'(L)) N(\frac{t}{\sigma(L)}) dt \mp \sigma'(L) \int_{-\infty}^{0} (f(L) \pm tf'(L)) \frac{t^2}{\sigma^2(L)} n(\frac{t}{\sigma(L)})) dt$$

the terms without a +/- sign in front cancel out in the sum $\Delta q = \Delta q_1 - \Delta q_2$ leaving

$$\Delta q = 2f'(L) \int_{-\infty}^{0} tN(\frac{t}{\sigma(L)}) dt - 2\sigma'(L) \frac{f(L)}{\sigma^2(L)} \int_{-\infty}^{0} t^2 n(\frac{t}{\sigma(L)})) dt$$

we can now change variables and get

$$\Delta q = 2f'(L)\sigma^2(L) \int_{-\infty}^{0} yN(y) dy - 2\sigma(L)\sigma'(L) f(L) \int_{-\infty}^{0} y^2 n(y) dy$$

The two integrals have the (exact) values respectively (integrate by parts)

$$\int_{-\infty}^{0} yN(y) dy = -\tfrac{1}{4} \text{ and } \int_{-\infty}^{0} y^2 n(y) dy = +\tfrac{1}{2}$$

so at last, remembering that $x = \mu = L + t$ we obtain

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[9] It is not actually normal for any given m but in fact we can check how non – normal the unsystematic risk distribution is in the circumstances of Basel 2001. This is done in the section on numerical work below.

[10] One can check at this stage that as claimed all the approximants in the integrand are negligible far from t = 0, just like the corresponding exact terms.

$$\Delta q = -\frac{1}{2}\frac{df}{d\mu}(L)\sigma^2(L) - \frac{1}{2}\frac{d\sigma^2}{d\mu}(L)f(L)$$

which is equation (9).

Derivation of Equation (2)

Let us now compute equation (4) for the CREDITRISK+ model used to derive the granularity adjustment in the New Accord. We will only show the result for a homogeneous portfolio with *m* obligors, using equation (6), because this is what is comparable with the work in the New Accord, Equation (4) does not assume homogeneity and the calculation for an arbitrary portfolio is similar to that presented below. It is not given here to save space.

The New Accord has default probabilities $P_A(x) = p_A(1-\omega_A+\omega_A X)$, where X is gamma with mean 1 and standard deviation $\sigma = 2$, or equivalently parameters $\alpha = 0.25$, $\beta = 4$ (Basel 2001, paragraph 445). ω_A is the "factor loading" (Basel 2001, paragraph 446). The 99.5% percentile of X has the value 12.007 (in *EXCEL*, this is *GAMMAINV(0.995, 0.25,4)*). To apply equation (6) we only need to consider a single obligor with exposure 1 and we have[11]

$$\mu(x) = LGD \times p(1-\omega+\omega x) \text{ and } \sigma^2(x) = (LGD^2 + VLGD^2) \times p(1-\omega+\omega x) \qquad (10)$$

In the IRB approach the volatility of loss given default $VLGD$ is defined as a proportion of exposure at default (Basel 2001, paragraph 447) as $VLGD = 0.5\sqrt{LGD(1-LGD)}$. Substituting in we get

$$\sigma^2(x) = LGD(0.25+0.75LGD) \times p \times (1-\omega+\omega x)$$

On differentiating

$$\frac{d\mu}{dx} = LGD \times p\omega \quad \text{and} \quad \frac{d\sigma^2}{dx} = LGD(0.25+0.75LGD) \times p\omega$$

Also we need the logarithmic derivative of the density function of the gamma distribution at the 99.5% point 12.007. Since $f(x) \propto x^{\alpha-1}e^{-x/\beta}$ and given that $\alpha = 0.25$ and $\beta = 4$ we have:

$$\frac{f'}{f} = \frac{\alpha-1}{x} - \frac{1}{\beta} = \frac{-0.75}{12.007} - 0.25 \cong -0.3125$$

Thus using (6) we get

$$\beta = \frac{-1}{2LGDp\omega}(-0.3125\sigma^2 + \frac{d\sigma^2}{dx})$$

which is

$$\beta = -1/(2\omega) \times (0.25+0.75LGD)(-0.3125 \times (1+11.007\omega)+\omega)$$

After some simplification and scaling the first bracket to look the same as the New Accord, this is

$$\beta = (0.4+1.2LGD)(0.762+0.098/\omega)$$

We now replace the factor loading parameter ω. The New Accord (Basel 2001, paragraph 446) shows that $F/PD = (X_{99.5\%}-1)\omega = 11.007\omega$ is the equation required to calibrate the CREDITRISK$^+$ and Vasicek models together. Using this we calculate

$$\beta = (0.4+1.2LGD)(0.762+0.098 \times 11.007\frac{PD}{F}) = (0.4+1.2LGD)(0.762+1.075\frac{PD}{F})$$

which is equation (2).

Note on inhomogeneous portfolios

The New Accord extends its approach to inhomogeneous portfolios by replacing them with a "hypothetical" homogeneous portfolio having the same amount of unsystematic risk (Basel, 2001, paragraph 452). However, since equations (4) to (6) are also valid for inhomogeneous portfolios

[11] These are first order in the default probabilities *p*. The exact formulae are well known but are given for example in Wilde (2001). (Equations 3 and 4). When trying to replicate numerical results obtained using CREDITRISK$^+$ one should keep the first order assumption because it is intrinsically used by that model.

(having different exposure sizes and / or PD's), one can follow the analysis above to calculate the theoretical inhomogeneous adjustment directly, without this extra step. The formulae obtained are not equivalent to the method chosen by Basel. Numerical work is needed to determine which formulae are more accurate.

Derivation of Equation (3)

Next we sketch the derivation in the case of the Vasicek model used for the base risk weights. In the Vasicek model, and in the derivation of the Base Risk Weights (Basel, 2001, paragraph 172) the specific dependence on the factor *X* is

$$P_A(X) = N\left(\frac{N^{-1}(p_A) + \rho_A^{1/2} X}{(1-\rho_A)^{1/2}}\right) \tag{11}$$

where X is now standard normally distributed i.e. $f_X(x) = n(x)$. In the New Accord the parameters are $\rho = 20\%$ and $X = 2.57$, its 99.5% value. As before we use formula (6), namely:

$$\beta = \frac{-1}{2f_X}\frac{d}{dx}\left(\frac{f_X\sigma^2}{d\mu/dx}\right) = \frac{-1}{2f_X}\left(\frac{d(f_X\sigma^2)/dx}{d\mu/dx} - \frac{f_X\sigma^2 d^2\mu/dx^2}{(d\mu/dx)^2}\right)$$

or

$$\beta = \frac{-1}{2}\left(\frac{\sigma^2 d(\log f_X)/dx}{d\mu/dx} + \frac{d\sigma^2/dx}{d\mu/dx} - \frac{\sigma^2 d^2\mu/dx^2}{(d\mu/dx)^2}\right)$$

Seeing that $d(\log f_X(x))/dx = -x$, this is

$$\beta = \frac{-1}{2}\left(\frac{-x\sigma^2}{d\mu/dx} + \frac{d\sigma^2/dx}{d\mu/dx} - \frac{\sigma^2 d^2\mu/dx^2}{(d\mu/dx)^2}\right)$$

As in the CREDITRISK$^+$ calculation above, we can put a portfolio consisting of a single obligor into the equation. In place of equation (10) we have

$$\mu(x) = LGD \times P_A(x) \text{ and } \sigma^2(x) = (LGD^2 + VLGD^2) \times P_A(x)$$

On rearrangement we get

$$\beta = \frac{-(LGD^2 + VLGD^2)}{2LGD}\left(1 - \frac{P_A}{dP_A/dx}\left(x + \frac{d^2P_A/dx^2}{dP_A/dx}\right)\right)$$

On differentiating (11) we get:

$$\frac{dP_A}{dx} = \frac{\rho_A^{1/2}}{(1-\rho_A)^{1/2}} n\left(\frac{N^{-1}(p_A) + \rho_A^{1/2}x}{(1-\rho_A)^{1/2}}\right)$$

and

$$\frac{d^2P_A}{dx^2} = -\frac{\rho_A(N^{-1}(p_A) + \rho_A^{1/2}x)}{(1-\rho_A)} n\left(\frac{N^{-1}(p_A) + \rho_A^{1/2}x}{(1-\rho_A)^{1/2}}\right)$$

so

$$x + \frac{d^2P_A/dx^2}{dP_A/dx} = x - \frac{\rho^{1/2}(N^{-1}(p) + \rho^{1/2}x)}{(1-\rho)^{1/2}} = \frac{x((1-\rho)^{1/2} - \rho) - \rho^{1/2}N^{-1}(p)}{(1-\rho)^{1/2}}$$

We therefore obtain

$$\beta = \frac{-(LGD^2 + VLGD^2)}{2LGD}\left(1 - N\left(\frac{N^{-1}(p) + \rho^{1/2}x}{(1-\rho)^{1/2}}\right)\frac{x((1-\rho)^{1/2}/\rho^{1/2} - \rho^{1/2}) - N^{-1}(p)}{n((N^{-1}(p) + \rho^{1/2}x)/(1-\rho)^{1/2})}\right)$$

We now substitute the Basel assumptions $VLGD = 0.5\sqrt{LGD(1-LGD)}$ as we did in the CREDITRISK$^+$ case above and the Basel parameters $\rho = 20\%$ (Basel, 2001, paragraph 172) and *X* = 2.57, its 99.5% value. After scaling the first factor to look like Basel's $0.4 + 1.2LGD$ we get:

$$\beta = (0.4 + 1.2LGD)\left(-0.3125 + \frac{N(1.118N^{-1}(p) + 1.288)(1.2499 - 0.3125N^{-1}(p))}{n(1.118N^{-1}(p) + 1.288)}\right) \tag{12}$$

The second factor in equation (12) is very different in form to the corresponding equation for CREDITRISK$^+$, but we may do a linear fit to *PD/F* where as above *F* is the systematic risk sensitivity. The function is very linear in the range 0.03% < *PD* < 10.00% and linear regression in that interval gives the coefficients

$$\beta^{Vasicek}_{Theoretical} = (0.4 + 1.2LGD)(0.29 + 4.29\frac{PD}{F})$$

with R-squared = 99.95%. This is equation (3).

Numerical verification of Equation (3)

It is prudent to use numerical work to verify that equation (3) is indeed a better estimate of the granularity adjustment for a Vasicek modelled portfolio. In the table below, I show the granularity adjustment, calculated both using equations (1) and (3) and numerically. I have chosen the portfolio with 200 obligors corresponding to Chart 5 in the New Accord (Basel, 2001, Chart 5).

N = 200	Granularity Adjustment (%)			
Default	CREDITRISK+		Vasicek	
Probability	Eqn 1	Numerical	Eqn 3	Numerical
0.10%	0.42%	0.38%	0.30%	0.30%
1.00%	0.45%	0.44%	0.40%	0.42%
2.50%	0.47%	0.46%	0.48%	0.51%
6.00%	0.50%	0.50%	0.63%	0.65%
15.00%	0.59%	0.56%	0.95%	0.94%

The CreditRisk$^+$ adjustments are those presented in the New Accord. (The actual figures are not given, but they can easily be read off from Chart 5 where they are the right hand most points[12]). As can be seen from the above chart, the numerical results for CreditRisk$^+$ do indeed agree closely with the results obtained using Equation (1) (the numerical results are the ones actually presented by Chart 5, I believe).

Likewise the Vasicek results above, obtained using my Equation (3), show good agreement with the corresponding numerical results. This serves to numerically verify Equation (3).

But as predicted, the Vasicek results and the New Accord's CreditRisk$^+$ results are rather different with the Vasicek results showing a much stronger dependence on the default probability. In this sense, then, the granularity adjustment as presented in the New Accord is inaccurate and the belief expressed in its derivation, that the change to CreditRisk$^+$ from the Vasicek model used to calibrate the base risk weights will not affect the results, is seen to be unjustified. If any coefficientshave the right to stand in paragraph 456 of the New Accord they are 0.29 and 4.29, not 0.76 and 1.1.

[12] It is worth noting that both the Basel (equation 1) and Vasicek (equation 3) granularity adjustments are accurate (relative, of course, to simulations using the CreditRisk$^+$ and Vasicek models respectively) for values of N down to as low as 10 – 20, well off to the right of Chart 7 in Basel, 2001.

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Why is the formulaic approach overly conservative ?

Let us consider the following simplified example where the securitised pool is comprised of three assets :

Asset	Value	Yearly PD	Capital charge under IRB
A	10	0.1%	4.68%
B	25	0.7%	15.96%
C	15	0.3%	9.32%

The total IRB charge on this portfolio is 5.856 [100% LGD is assumed] for the year coming.

Assuming independence in default between these three assets [maximises the probability of first loss], we obtain the following loss distribution :

Loss	Probability
10	.1% x 99.7% x 99.3%
15	.3% x 99.9% x 99.3%
10+15	.1% x.3% x 99.3% +.7% x 99.7% x 99.9%
10+25	.1% x .7% x 99.7%
15+25	.3% x .7% x 99.9%
10+15+25	.1% x .7% x .3%

The probability of losing at least 10 [first loss] is the sum of all the probabilities above, close to the sum of the individual yearly PDs on each of the three assets (1.1%) The IRB charge for this exposure is 2.12.
The probability of losing an additional 5 is 1%, yielding an additional IRB charge of 1.

Using this approach, we can assign PDs and corresponding IRB charges to all tranches of loss on the loss distribution. The corresponding global IRB charge is comparable to that on the original portfolio, certainly not higher, as proposed in the Basel Committee's draft (5.28).

If first loss is everything not investment grade (the first 15 lost), as purported in the Committee's draft, then it attracts a conservative (independence having been assumed) capital charge of 3.12, far from the proposed $ for $ capital.

<u>Further questions worth addressing :</u>

Is a formulaic approach required ?

If yes, could it be based on an assumption of default independence across counterparties in the pool (see above) ? Or on the use of the rating agencies' models' correlation assumptions ?