FRTB IMA DRC and the 3 Basis Point Floor

November 2022
In the Fundamental Review of the Trading Book (FRTB), the Basel Committee on Banking Supervision has set a floor of 3 basis points (bp) for the probability of default (PD) of any entity in the default risk charge (DRC) of the internal models approach (IMA). The floor is not applicable to the standardized approach (SA) DRC and is not applied to the IMA of sovereign PDs under the internal-ratings-based (IRB) approach.

This paper sets out statistical, data-driven research to explain why it is not necessary or appropriate for this floor to be applied to the IMA DRC. When the study is extended to covered bonds, it is not possible to reach a statistically significant conclusion due to the sample size, but there is no reason why this asset class would diverge from the broader conclusions in the paper.
## MAIN FINDINGS AND RECOMMENDATIONS

### Key Messages

- Sovereign bond markets are among the largest and most important financial markets in the world. It is vital there is liquidity in this market, and this is facilitated by the banks that trade sovereign bonds.

- Since rating agencies first began rating sovereigns until the end of 2021, no sovereign issuer has ever defaulted after starting the year with an investment-grade rating.

- In the current FRTB framework, the treatment of the IMA DRC PD floor, the SA DRC and the IRB floor is inconsistent. The 3bp floor does not apply to sovereigns as part of the SA DRC or the IRB. Sovereign exposures should not lead to higher capital consumption for the IMA DRC than for the IRB.

- The 3bp floor would have the biggest impact on AAA-rated sovereigns, as they are expected to have the lowest default probability. The application of Bayesian inference to estimate the PD for AAA-rated sovereigns based on historical data for corporate bonds (spanning 1981–2021) and for sovereigns (spanning 1990–2021) shows the mean of the distribution is below 1bp.

- Sensitivity analysis shows that the results are not overly sensitive to any one idiosyncratic event that might occur in the future, such as the default of Russia (BBB-rated) in early 2022.

- There is no instance of a rated covered bond that has defaulted after starting the year with a specific rating category. Due to the nature and relative size of this market, it was not possible to apply the Bayesian inference model to estimate with a reasonable level of significance, but there is no reason to believe the key conclusions of this paper should be different for this low-risk asset class.

- Comparison of the effect of a 3bp floor in the IRB approach to the effect of the same floor in a DRC model shows that the impact of the floor can be more than three times greater on the DRC than on the IRB. However, there is no floor applied to sovereigns under the IRB approach.
BACKGROUND

THE SOVEREIGN BOND MARKET

Sovereign bonds are issued by governments to finance investment and grow their economies. Governments borrow domestically but also depend on access to foreign funding. Sovereign debt can be issued in both domestic and foreign currencies. In 2021, the total global amount of central government debt outstanding was $29,066.8 billion, according to the Bank for International Settlements\(^1\).

It is essential that governments continue to service their debt and their debt burden remains sustainable. As sovereign bonds are issued by national governments, they are generally considered among the safest investments. A unique characteristic of sovereign debt is that governments can generate tax revenue while simultaneously printing their own currency. Sovereign entities have some degree of influence over interest rates, have an independent central bank and have access to the marketplace via investors that will purchase sovereign bonds. In comparison, corporate bonds are issued by highly idiosyncratic companies that may be more likely to falter based on many factors.

Banks help to facilitate the link between borrower and lender, but they also have other incentives to participate in the sovereign debt market. Sovereigns are useful for a bank’s balance sheet management – specifically, liquidity management. In many jurisdictions, sovereign debt instruments are among the most liquid assets and are therefore suitable for use as collateral. They also play a role in market making as many banks hold sovereign debt as part of their role as primary dealers or market makers for such exposures.

Sovereigns can be seen as offering an attractive risk-return investment. For example, banks may rebalance their portfolios during downturns and favor sovereign exposures relative to other investments. The current risks that do exist in the global sovereign bond market are captured through credit spreads.

In summary, banks hold sovereign bonds for different purposes, often linked to liquidity management or their role as a primary dealer or market maker. A 3bp floor will increase capital requirements for banks that hold high-quality sovereign debt, which may impact liquidity in those markets. This could, in turn, impact funding costs for highly-rated sovereigns. Application of the floor to sovereigns would impede banks’ ability to warehouse sovereign debt in instances when other entities, such as pension funds, are required to unload their positions into the market. Imposing a PD floor for instruments considered by many to be risk-free would certainly have a negative impact.

\(^1\) Debt securities statistics, Bank for International Settlements, [www.bis.org/statistics/secstats.htm](http://www.bis.org/statistics/secstats.htm)
IMA DEFAULT RISK CHARGE

The FRTB is an international standard designed by the Basel Committee on Banking Supervision to set a framework for the capital banks must hold against their market risk exposures. As part of the FRTB, the DRC is a calculation designed to capture the default risk of credit and equity trading book exposures, with no diversification effects allowed with other market risks. Firms may use either a regulatory-prescribed standardized model or an internal default risk model.

The IMA DRC is a replacement for the incremental risk charge (IRC) introduced in Basel 2.5, and it works in a similar way.

However, there are three key differences:

- The DRC covers default risk only, whereas the IRC covered default and migration risk;
- The DRC must include equity and debt positions, whereas the inclusion of equity was optional in the IRC; and
- The DRC includes a 3bp floor on the PD inputs to the model for all issuers.

This paper focuses on the last of these three components – the PD floor – and examines whether this constraint is appropriate or proportionate.

THE 3BP PD FLOOR

The IMA DRC requires firms to use PDs based on a one-year time horizon, which should be based on historical data. Furthermore, in the EU, UK and US, the model-generated PDs in the IMA DRC and the Basel 2.5 IRC have the same interpretation as the model-generated PDs in the IRB approach (for firms with permission to use internal models to calculate credit risk capital requirements).

The IRB approach requires PDs to be floored at 3bp before calculating credit risk capital requirements, and this appears to be the origin of the 3bp floor in the DRC. However, the PD floor in the DRC is applied to the PD for all issuers, while the floor in the IRB does not cover sovereign issuers (central banks and governments). Furthermore, the PD for investment-grade sovereigns when calculated based on historical defaults using conventional methods is zero. Importantly, the floor affects the methodology in the IRB and IMA differently.

Additionally, while the FRTB-IMA imposes a PD floor of 3bp, no such floor is imposed as part of the SA DRC. This imposes a boundary on internal models, so that even the most creditworthy country would not be permitted to use a PD under 3bp. This has a significant impact on capital and undermines the internal models used in the DRC.
USE OF BAYESIAN METHODS TO ESTIMATE AAA-RATED SOVEREIGN PD

Since rating agencies first began rating sovereigns until the end of 2021, no sovereign issuer has ever defaulted after starting the year with an investment-grade rating. A lack of default data can pose challenges for the estimation of default probabilities using conventional tools. Bayesian methods are often seen as a suitable option in these cases for the estimation of PDs for low-default probabilities. Bayesian inference is a method of statistical inference in which Bayes’ theorem is used to update the probability for a hypothesis as more evidence or information becomes available.

In this paper, Bayesian techniques are applied in a model that generates a full probability distribution for the PD of each rating class/issuer type combination (eg, AAA-rated corporates versus AAA-rated sovereigns), conditional on the observed empirical data and two key constraints:

1) The ordinal ranking of ratings is correct, so the PD for one rating class and issuer type must be lower than the PD for a lower rating class and the same issuer type.

2) Corporates have a higher PD than sovereigns of the same rating class.

The model was used to estimate the distribution of PDs for each combination of rating class/issuer type, conditional on the empirical data in Table 1, for a range of input correlations. Figure 1 shows the mean and 99th percentile of the PD distribution for AAA-rated sovereigns. For default correlations up to 50%, the mean of the PD distribution is below 1bp. At a correlation of 32%, the mean PD is ~0.6bp. The distribution of possible values of the PD is centered around the mean. The 99th percentile falls just below the proposed floor of 3bp (ie, 99% of possible values of the PD are expected to be below 3bp).

SENSITIVITY TO IDIOSYNCRATIC EVENTS

The historical data used for this study covers defaults up until the end of 2021. However, Russia defaulted in 2022, having been rated BBB at the start of the year – higher than any sovereign has ever been rated by S&P at the start of a year in which it defaulted. To understand the sensitivity of the results to idiosyncratic events, a sensitivity analysis was carried out and the model was re-run multiple times, with one extra sovereign default to a single rating category added each time.

Figure 2
Change in the Mean of the Distribution of AAA-rated Sovereign PDs When an Additional Default is Included in the Dataset

Note: The chart shows the impact on the estimated PD to AAA-rated sovereigns when an additional default of the same or lower rating categories is included in the dataset
Source: Results of sensitivity analysis using Bayesian Inference Model

Figure 2 shows the results of the analysis, which was carried out using a default correlation of 32%\(^3\). Including the default of Russia (rated BBB) would have had a negligible impact on the mean of the PD distribution for AAA-rated sovereigns. Even an AAA-rated sovereign default, were one to occur, would not increase the mean above 1.5bp. The results are therefore not overly sensitive to any one idiosyncratic event that might occur in the future.

EXTENSION TO COVERED BONDS

The analysis in this paper considers only two distinct types of bonds – unsecured corporate debt and sovereign-issued debt. A third category is covered bonds. In the S&P dataset, there has never been a recorded default on a covered bond. This is in line with economic intuition, as holders of covered bonds have dual recourse (to the asset pool and then to the bond issuer), meaning investors only face a loss if the asset value collapses \textit{and} the issuer defaults. However, the Bayesian inference model is not suitable for application to covered bonds due to the low number of data points. The low risk of AAA-covered bonds, together with empirical data, indicates that the conclusion for sovereigns should also be relevant for covered bonds.

DRC IMA VS IRB COMPARISON

As previously highlighted, the 3bp PD floor in the IMA DRC appears to originate from the equivalent 3bp floor in the IRB credit model, although this is not applied to sovereign issuers in the IRB. The IRB credit model is an asymptotic single risk-factor default risk model, calibrated using one-year probabilities of default and to a 99.9th percentile, which is the same as the DRC. As a result, it might be assumed that the impact of applying a floor to the input PDs for the DRC would have a comparable impact to applying the same floor to the input PDs for the IRB.

This is not the case, however, and the main reason for this is the correlation structure\textsuperscript{4}. The IRB model, as a single risk-factor model, necessarily imposes a very simple correlation structure. All issuers are correlated only via their correlation to the systemic factor, and that correlation ranges from 12\% to 24\%, with decreasing probability of default\textsuperscript{5}. DRC models, on the other hand, are required to use at least two types of systematic risk factors, which impose a richer correlation structure.

To illustrate the difference that a non-homogeneous correlation structure would make, Figure 3 shows the impact of the 3bp floor on an IRB model and a very simple DRC model (see Methodology and Evidence section for further details of the assumptions and portfolio used).

\textsuperscript{4} The IRB also incorporates an element of migration risk through a maturity factor adjustment. Here, all maturities are set to one year to remove this effect in the comparison

\textsuperscript{5} For large financial entities, the correlation ranges from 15\% to 30\%
Figure 3
Impact of the 3bp Floor on DRC and IRB Treatment

Note: The chart shows the impact of the 3bp floor on the DRC and IRB treatment for a portfolio of 25,000 AAA-rated bonds split into developed and emerging market (EM) segments, varying the correlation of the EM segment while holding the portfolio average correlation constant at 24%.

Source: Results of IRB and simplified DRC model constructed to perform the analysis.
METHODOLOGY AND EVIDENCE TO SUPPORT THE REMOVAL OF THE 3BP FLOOR

EMPIRICAL DEFAULT DATA

Table 1 shows empirical default statistics extracted from S&P CreditPro®. For each rating category, data is provided for the number of issuers that entered each year at a particular rating (N) and the number of defaults within that rating category during that year (D), aggregated across multiple years.

Table 1: Historical Default Data Per Rating Category

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The historical default data is aggregated across years. The data has been extracted from S&P CreditPro® and reaches the end of 2021. Data for corporate bonds spans the interval from 1981 to 2021, while data for sovereign bonds spans the interval from 1990 to 2021.
Since rating agencies first began rating sovereigns, the data shows no defaults have been recorded for countries that started a year with an investment-grade rating (ie, BBB or higher). There were also no defaults recorded for countries that started the year at the investment-grade ratings of B+, BB+ and BBB-.

**PD CALIBRATION**

Issuers are assigned a credit rating based on an assessment of their riskiness and likelihood of default. In line with standard practice, it should be possible to derive an estimate of default probability from the credit rating.

However, it is not straightforward to calibrate an estimate of PD to the historical data for rating categories in which there were no observed defaults. The maximum likelihood estimator (MLE) of PD for a rating category with $N$ observations and $D$ defaults is given by $D/N$. Using this method and the empirical data in Table 1, it can be observed that the MLE PD for all investment-grade sovereign rating classes is zero. Although this is the correct MLE PD, it is also necessarily a lower bound on the PD. The fact that there have been no observed investment-grade sovereign defaults does not mean there is no possibility that such an event will occur in the future. It is therefore necessary to consider methods that probe further than the MLE of the PD.

**BAYESIAN MODEL**

*Methodology*

Bayesian methods are often seen as a suitable option for PD estimation for situations where the observed frequency is small or zero. Bayesian inference is a method of statistical inference in which Bayes’ theorem is used to update the probability for a hypothesis as more evidence or information becomes available.

Bayesian techniques can be used to estimate PDs for corporate and sovereign issuers. Here, a method is described based on a model proposed by Chourdakis and Jenna⁶. The model generates a full probability distribution for the PD of each rating class/issuer type combination (eg, AAA-rated corporates versus AAA-rated sovereigns), conditional on the observed empirical data and two key constraints.

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These constraints are:

1. The ordinal ranking of ratings is correct, so the PD for one rating class and issuer type must be lower than the PD for a lower rating class and the same issuer type. For example, the PD for AAA-rated sovereigns must be lower than the PD for AA+ sovereigns, and the PD for BBB-rated corporates must be lower than the PD for BBB- corporates. This constraint is consistent with the basic purpose of credit ratings – that they can be used to separate entities based on their PDs.

2. Corporates have higher PDs than sovereigns of the same rating. For example, the PD of an AA+ corporate must be higher than the PD of an AA+ sovereign. This constraint reflects the different nature of sovereign default events, which are typically played out over a longer time (allowing rating agencies to downgrade them on the way), as opposed to corporate defaults, which can be idiosyncratic and occur faster. This relationship is also visible in the empirical data.

The prior PD for each rating class/issuer type combination (ie, the PD, unconditional on any of the empirical data) is also set to be uniform across the region permitted by the two constraints.

The result of these assumptions is that the probability distribution for the PD of any single rating/issuer combination depends not only on the empirical data for that rating/issuer, but on all the empirical data consumed by the model. For example, every time an AAA-rated sovereign is observed to survive within a year, it will lower the estimated PD for AAA-rated sovereigns. But every time an AA-rated sovereign is observed to survive, or an AAA-rated corporate is observed to survive, that will also lower the estimated PD for AAA-rated sovereigns, as the PD for AAA-rated sovereigns is constrained to be lower than the PD for AA-rated sovereigns and AAA-rated corporates.

Conditional on a set of empirical data, there is no analytic solution to find the probability distributions for PDs based on these assumptions. However, numerical methods can be used as a viable alternative. Specifically, the Metropolis-Hastings algorithm – a Markov Chain Monte Carlo (MCMC) method – can be used.

A solution space is defined by the set of PDs for each rating class/issuer type combination that is permitted under the two constraints. For any two points in the solution space, a likelihood ratio conditional on the observed market data can be calculated.
A random walk through the solution space is executed by generating perturbations to the set of PDs and calculating the likelihood ratio between the candidate point and the current point. If the likelihood ratio is greater than one (ie, the observed data would be more likely to have arisen from the candidate set of PDs than from the current set of PDs), then a step is taken to the candidate. If the likelihood ratio is less than one (ie, the observed data would be more likely to have arisen from the current set of PDs than from the candidate set of PDs), then a step is taken to the candidate with probability equal to the likelihood ratio. As the number of steps increases, the path traced through the solution space converges to a set of samples drawn from the (stationary) posterior distribution. In other words, an empirical probability distribution for the set of PDs is built.

The model is run with two million steps and uses the augmented Dickey-Fuller test to demonstrate the set of samples has converged to a stationary distribution.

The model must be adapted to account for correlation. In any given year, defaults are not independent events and are often clustered because of systemic macroeconomic factors (or direct causation). Although this does not change the MLE PD, it does affect the likelihood ratio between two potential PDs. For example, if a BB-rated corporate default is observed every year from 1980 to 2020, this will lead to the posterior distribution of the BB-rated corporate PD being updated to reflect the higher probability of default. But if 40 BB-rated corporates default in a single year, then the presence of default correlation means this is less statistically significant and so the impact on the posterior distribution will be less.

Calculating the likelihood ratio in the presence of default correlation is not possible analytically (and not practical numerically inside a two-million-scenario MCMC simulation). However, the effect of default correlation⁷ can be mimicked by applying a scaling parameter to the empirical data.

The impact of this correlation can be quantified by considering the following proposition. Consider a collection of $N$ reference credits with a common PD, $p$, and Gaussian correlation, $\rho$. The number of defaults is given by $D$. Then

$$E\left[\frac{D}{N}\right] = p$$

$$V\left[\frac{D}{N}\right] = \frac{p - p^2}{N} + \left[G(p,\rho) - p^2\right] \frac{N - 1}{N}$$

where

$$G(p,\rho) = \Phi_2(\Phi^{-1}(p), \Phi^{-1}(p); \rho)$$

$\Phi^{-1}$ and $\Phi_2$ denote the univariate and bivariate Gaussian CDF, respectively

Given this proposition, the mean and variance of $D/N$ is the same as the mean and variance of $D_0/N_0$, where $D_0$ is the number of defaults out of a population of $N_0$ uncorrelated credits that exhibit the same PD. The effective sample size $N_0$ satisfies:

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⁷ Scaling down the empirical data by this scaling factor can be shown to produce a distribution where the first two moments of $D/N$ (scaled, treated as independent) match the first two moments of $D/N$ (unscaled, correlated)
\begin{align*}
    N_0 &= \frac{1}{N} \left( 1 + (N - 1) \frac{G(p, \rho)}{p(1-p)} \right)
\end{align*}

For each year, the number of observations, \( N \), and the number of defaults, \( D \), are both scaled down by a constant equal to:

\[
1 + (N - 1) \frac{\Phi_{\text{bivariate}}(\Phi^{-1}(p), \Phi^{-1}(P); \rho) - \rho^2}{p(1-p)}
\]

where \( \Phi \) is the standard normal density function and \( \Phi_{\text{bivariate}} \) is the standard normal bivariate density function, \( \rho \) is the default correlation, and \( P \) is the probability of default\(^8\).

There is no straightforward way to estimate the correlation for events (such as the default of an investment-grade sovereign) that have never taken place, and the results of the model are presented for a range of different correlation values. For sovereigns, however, a correlation value of 32% is highlighted, which corresponds to the asset correlation used by Moody’s in its Gcorr 2019 model for sovereigns with credit default swap data. This is not dissimilar to the value of ~24% prescribed by regulators to use in the IRB model for issuers with low probabilities of default.

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\(^8\) The scaling factor depends on the probability of default, \( P \), which is an output of the model. The model is initially run using \( P \) equal to \( N/D \) for each rating class/issuer type combination, aggregated across all years. \( P \) is then updated to be the mean of the estimated PD distribution for each class and the model is repeated, iterating until the input value of \( P \) is consistent with the output mean.
Results

The model was used to estimate the distribution of PDs for each combination of rating class/issuer type, conditional on the empirical data in Table 1, for a range of input correlations. Figure 4 shows the mean of the PD distribution for AAA-rated sovereigns. For default correlations up to 50%, the mean of the PD distribution is below 1bp. At a correlation of 32%, the mean PD is ~0.6bp. The 99th percentile is slightly lower than the proposed floor of 3bp.

Figure 4
99th Percentile and Mean for the PD of AAA-rated Sovereigns for Range of Correlations

Note: The chart shows the estimated PD for a range of correlations. 32% is the correlation suggested by Moody’s Gcorr model for sovereigns that have credit default swap spreads
Source: Results of Bayesian Inference Model

Sensitivity Analysis

The historical data used for this study covers defaults up until the end of 2021. However, Russia defaulted on its debt in early 2022 and was rated BBB at the start of the year – higher than any sovereign (rated by S&P) at the start of a year in which it defaulted. To understand the sensitivity of the results to idiosyncratic events, a sensitivity analysis has been carried out and the model was re-run multiple times, with one extra sovereign default to a single rating category added each time.
Figure 5 shows the results of the analysis, carried out using a default correlation of 32%. Including the default of Russia (rated BBB) would have a negligible impact on the mean of the PD distribution for AAA-rated sovereigns. Even an AAA-rated sovereign default, were one to occur, would not increase the mean above 1.5bp. The results are therefore not overly sensitive to any one idiosyncratic event that might occur in the future.

Impact of Different Start Dates for the Year Periods

The data used in the analysis covers default events applicable to a rating category, where an entity has started the calendar year at that rating and the default has occurred before December 31. IMA DRC PDs are based on a one-year horizon, so the rating should be considered in the 12 months prior to the default event, which may not fall in the same calendar year.

To consider the impact of the calendar assumption on the overall results, the analysis was re-run using different quarterly start dates for the one-year period. The results show there is very little impact from the change in start date (see Figure 6). The impact of assuming defaults in a calendar year that starts on January 1 therefore has a negligible effect on the overall results.
EXTENSION TO COVERED BONDS

The quantitative analysis in this paper considers two distinct types of bonds—unsecured corporate debt and sovereign-issued debt. A third category is covered bonds. Table 2 shows the historical default dataset from CreditPro® with covered bonds included.

Figure 6
Impact of Choice of Start Date for Yearly Time Periods

Note: The chart shows the impact on the estimated PD when using different quarterly start dates for each yearly period
Source: Results of sensitivity analysis using Bayesian Inference Model
Table 2: Historical Default Data Including Covered Bonds

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</table>

The historical default data is aggregated across years for unsecured corporate/sovereign/covered bond issuers.

In the S&P data, there has never been a recorded default by a covered bond. This is in line with economic intuition, as holders of covered bonds have dual recourse – to the asset pool and then to the bond issuer. This means investors can only face a loss if both the asset value collapses and the issuer defaults. However, the Bayesian inference model is not suitable for application to covered bonds due to the low number of data points. The low risk of AAA-rated covered bonds, together with empirical data, indicates that the conclusion for sovereigns should also be relevant for covered bonds.

**IMA DRC VS IRB COMPARISON**

As previously highlighted, the 3bp PD floor in the IMA DRC model appears to originate in the equivalent 3bp floor applied in the IRB credit model (although it is not applied to sovereign issuers in the IRB). The IRB credit model is an asymptotic single risk-factor default risk model, calibrated using one-year PDs and to a 99.9th percentile (which is the same as the DRC). As a result, it might be assumed that the impact of applying a floor to the input PDs for the DRC would have a comparable impact to applying the same floor to the input PDs for the IRB.
However, this is not the case, primarily because of the correlation structure. The IRB is a single risk-factor model, which necessarily imposes a very simple correlation structure. All issuers are correlated only via their correlation to the systemic factor, with the correlation ranging from 12% to 24% with decreasing PD. DRC models, on the other hand, are required to use at least two types of systematic risk factors, which imposes a richer correlation structure.

Figure 7 shows the impact of the 3bp floor on an IRB model and a very simple DRC model. The models are run on a portfolio of 25,000 AAA-rated bonds divided into two segments – developed market (DM) issuers and emerging market (EM) issuers. The DM issuers are all uncorrelated, while the EM issuers are correlated via a systemic EM factor.

To begin with, all 25,000 bonds are from EM issuers and the EM factor loading is set to produce a 24% correlation, which matches the IRB correlation. In this scenario, the DRC model is effectively equivalent to the IRB model. As expected, the impact of the floor is the same on both models.

The EM factor loading is then increased and some of the EM bonds are simultaneously switched to DM bonds to keep the average portfolio correlation constant at 24%. As pockets of high correlation appear within the portfolio, these correlated names begin to drive the DRC output disproportionately. As a result, the sensitivity to the floor increases. By the time the EM bonds have a correlation of ~90% (and represent about one quarter of the portfolio), the 3bp PD floor has a three-times greater impact on the DRC than on the IRB.

9 The IRB also incorporates an element of migration risk, through a maturity factor adjustment. Here, all maturities are set to one year to remove this effect from the comparison.

10 For large financial entities, the correlation ranges from 15% to 30%.
While the results of this analysis are not surprising, they clearly illustrate that the richer correlation structure of the DRC model means the floor is likely to have a larger impact than it would on an IRB model, even if the average correlation across the portfolio matches the IRB correlation prescribed by regulators.
CONCLUSION

Based on historical default data for corporates and sovereigns from 1981 to 2021, Bayesian inference can be used to imply the probability distribution for the PD of different ratings and issuer types. Under mild assumptions, the mean of the distribution for AAA-rated sovereigns is below 1bp and 3bp is slightly more conservative than the 99th percentile (implying 3bp is in the tail of tail distribution). Based on this analysis, it is clear that a 3bp floor applied to AAA-rated sovereigns is arbitrary and highly conservative.

A key difference between sovereign and corporate debt is that governments do not generally default on all their obligations. More commonly, sovereigns selectively default on portions of their debt. The analysis presented in this paper assumes default on the whole set of commitments and still demonstrates that the 3bp floor is highly conservative.

The paper also shows the impact of a floor is likely to be much higher on a DRC model than an IRB model due to the richer correlation structure used in DRC models. If migration of the 3bp floor from the IRB to the DRC was intended to equalize the capital treatment of equivalent portfolios in the trading book and the banking book, the analysis suggests this will not occur and could in fact lead to arbitrage opportunities.
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In this note, we have supported ISDA by providing the analytical foundation of the findings, including the impact of the FRTB regulation and the PD Floor.