31st July, 2014

Ms. Norah Barger and Mr. Karl Cordewener, Co-Chairs, Trading Book Group
Basel Committee on Banking Supervision
Bank for International Settlements
Centralbahnplatz 2, CH-4002 Basel, SWITZERLAND

Sent by email to: norah.barger@frb.gov; karl.cordewener@bis.org; Juquan.Tan@bis.org; baselcommittee@bis.org

Re: Second Consultative Document Fundamental Review of the Trading Book[1] (CP2) – BCBS 265 – Quantitative Impact Study instructions – Supplementary note on SBA

Dear Ms. Barger and Mr. Cordewener,

This letter contains additional comments from the Associations[1] on the draft instructions received on 23rd June 2014 in relation to the quantitative impact study ("QIS"), launched by the Basel Committee on Banking Supervision ("BCBS") on the Fundamental Review of the Trading Book ("FRTB").

As previously communicated in the relevant response (dated 17th July 2014) the industry has identified a flaw in the prescribed delta risk aggregation formula. Having performed additional analysis, participants were able to identify a number of potential solutions that can mitigate the occurrence of negative variance in the aggregation formula of delta risk capital charges across buckets. Herein, our members outline these preliminary solutions and their pros and cons.

Considering the importance of the SBA calculation in the market risk capital framework and the daily calculation of the SBA capital charge over constantly changing portfolios once the FRTB framework is implemented, the Associations strongly advocate that this issue needs to be addressed and we would be happy to facilitate a discussion with the industry experts group in short notice.

We stress again our commitment to participate constructively in the consultative process and we do sincerely hope you find our input helpful.

Yours faithfully,

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1 Basel Committee on Banking Supervision, October 2013

[1] The International Swaps and Derivatives Association, Inc. ("ISDA"), the Global Financial Markets Association ("GFMA") and the Institute of International Finance ("IIF")

Mark Gheerbrant	David Strongin	Andres Portilla
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I. INTRODUCTION

The FRTB's standardised SBA prescribes (in paragraph 8) the following method for calculating the Delta Risk capital requirement from weighted sensitivities. First the weighted sensitivities WS_k within a given bucket b are combined to give the capital charge K_b for that bucket:

$$K_b = \sqrt{\sum_k WS_k^2 + \sum_k \sum_{l \neq k} \rho_{kl} WS_k WS_l}$$

Then the capital charges for the various buckets are aggregated as follows:

$$Delta\ Risk\ Charge = \sqrt{\sum_b K_b^2 + \sum_b \sum_{b \neq c} \gamma_{bc} S_b S_c} + K_{residual}$$

The second formula has the problem that **the term inside the square root can go negative**. We provide an example in appendix 1 that illustrates the occurrence of negative variance in the SBA risk aggregation formula.

II. PROBLEM DEFINITION

The industry fully appreciates the TBG's effort to make the SBA more risk sensitive, in particular by capturing hedging and diversification benefits through the introduction of a variance-covariance type of an approach for the aggregation of the capital charges for the various buckets. At the same time we comprehend the difficulty which the regulators are facing in i) specifying a large scale correlation matrix across all risk factors and ii) establishing a convenient and transparent cascading approach to the aggregation of capital across buckets.

The proposed methodology adopts a cascading approach with the aggregation of capital charges at the risk factor level, then at broader bucket level and finally at the asset class level. This approach has the benefit of requiring only the specification of a manageable number of correlations in each step. Moreover, the proposed regulatory formulas define two bucket level metrics (Kb and Sb) that have the following financial interpretation:

- **Kb** measures bucket level risk while capturing correlation and diversification across risk factors within the same bucket;
- **Sb** measures bucket level risk assuming no diversification across risk factors of the same bucket.

A valid K_b requires that the two correlation matrices (for same signs and different signs) meet the conditions for positive semi-definiteness as illustrated in Appendix 5.

The issue with the second formula is more complex and is the subject of the rest of this document.

When testing the prescribed formula, we identified inconsistencies in the outputs regarding capital numbers. We identified that this was caused by the use of Kb and Sb in the same variance-covariance-like aggregation formula for asset class level delta-risk charge. The source of the issue is that the risk correlations across buckets depend on the composition and particularly the directionality of the risk factor exposures within each bucket. Considering a simple example of only two buckets and only one risk factor within each bucket, the sign of the correlation between the risky P&L corresponding to the two buckets would depend on whether we are long both buckets or long one and short the other. This directional information is lost in the calculation of the metric Kb which is always positive if the correlations meet the conditions illustrated in appendix 5. As such, specifying the same correlations for the Kb's without further information about the composition/directionality of exposures within each bucket can lead to significant misrepresentation of asset class level risk and therefore to the capital allocation.

FRTB CP2 attempts to solve this issue by using Kb for the bucket level variance term while using Sb for the cross terms. The Sb being the sum of risk weighted exposures, which contains some information about directionality. This however creates another problem in the formula as the Sb terms represent bucket level risk assuming no risk factor level diversification, while the Kb's represent bucket level risk with risk factor level diversification. Consequently, there are situations whereby:

- The cross terms will dominate the direct variance terms and
- **Thereby** lead to a negative asset class level variance,
- **Hence** giving rise to a square-root of a negative variance during the calculation of delta risk charge at the asset class level.

The industry provided an example of this in our recent communication with the TBG (see Appendix 1).

Essentially, the aim of the example was to demonstrate that while it may seem easier and more transparent to use a cascading approach for specifying correlations, one cannot freely select correlations and use any variance-covariance-like aggregation. The "implied correlations" at the lowest risk factor level can be inconsistent and thus could lead to economically non-meaningful results, such as negative variance at the asset class level.

In order to avoid delays in the QIS, the industry has discussed a number of possible solutions to address this problem in the aggregation of capital charges for SBA, which we hope that the TBG will consider in its deliberations. In this context, we would like to note that given the very short time over which our members have put the proposals forward, we have not had the time to conclude on a preferred industry solution at this stage. We would like to discuss the options with the TBG in order to identify potential considerations and come to a conclusion regarding the most suitable option, given the objectives of the FRTB.

III. POSSIBLE SOLUTIONS

The possible solutions to address the occurrence of negative variance in the SBA capital calculation fall under four different approaches:

(1) Directly deal with the negative variance symptom by **capping/flooring the capital charge calculation**. Two solutions fall under this line of thought:

   a. Floor the term inside the square-root of the delta charge equation to 0. This is a "brute force" approach; it is more consistent with the current FRTB2 specification and always solves the negative variance problem. However, it does not deal with the root cause of the inconsistency issue and we still have two separate bucket level metrics (Kb and Sb) in the same equation.

   b. Cap the absolute value of Sb at Kb. This method solves the problem as long as the underlying correlation matrix (the rhos) is positive semi-definite. Similar to (a), it does not attempt to deal with the root inconsistency problem but it essentially imposes the constraint that bucket level risk with no diversification is smaller than bucket level risk with diversification.

(2) **Eliminate the two bucket level metrics**

   a. Use signed Kb instead of Sb in the delta risk charge equation. The methodology then reduces to a standard variance-covariance form and as long as the correlation matrix is positive semi-definite, there will not be a negative variance problem.

   Please find further analysis on options (1a), (1b) and (2) in Appendices 2 and 3.

(3) **Reduce the magnitude of the cross terms**

   a. The simplest way is to ignore the correlation across risk buckets i.e. assume zero correlation. While this can certainly eliminate the negative variance problem, it also disallows any hedging benefits across buckets, contradicting the intention for a more risk sensitive standardized approach.

   b. Make the cross buckets correlations (gamma_bc's) smaller but not zero or set them in a way (e.g. outer correlation equal to the product of corresponding inner correlations) to mitigate the problem.

(4) **Instead of using cross terms such as Sb*Sc, use cross terms that are consistent with correlation assumptions in the calculation of Kb**

   a. Together with a scaling factor between 0 and 1 of cross bucket correlation, this can mitigate the negative variance problem. The downside of this approach is that the cross terms should be calculated over all pairs of buckets and it's not modular as the other options.

   See Appendix 4 for further description of this approach.

Moreover the fact that FRTB CP2 specifies different correlations when exposures have the same sign versus exposures with different signs results to magnification of the problem of ensuring a positive definite correlation matrix and hinders the efficacy of the solutions presented above.

See Appendix 5 for a condition the same and different sign correlations have to satisfy in order to ensure the positive definiteness

IV. CONCLUSION

Considering the above and that the QIS is to start imminently, we conclude that there is not adequate time to work on re-specification/recalibration of the correlations across the different risk factors. Therefore, the only options we present that can be incorporated to ~~immediate QISs~~the current QIS (but can be amended in the subsequent QISs), would be the simplest ones such as (1) and (2) and (3a) which do not attempt to solve the root cause of the problem but do avoid the negative variance issue.

Elaborating further on these potential solutions (1a, 1b, 2a and 3a), option 1a is the most consistent with the FRTB CP2/SBA equation. Option 1a simply floors the term within the square-root while options 1b and 2a effectively alter the SBA equation. Option 1b uses a formula that produces a smoother and continuous capital estimate, while option 2a avoids having two bucket level metrics and therefore is closest to a standard variance-covariance formula. Option 3a disallows cross-bucket correlation so is less risk sensitive, but represents a very simple way to combine buckets. Considering the low level of gammas (cross buckets correlations) specified in FRTB/ CP2, the effect of ignoring cross buckets hedging could be relatively immaterial although more analysis is needed to validate this point.

Considering the limited time that was available for the analysis of this problem and the investigation of potential solutions, industry participants are expressing the need for more time in exploring these options and understanding the full implications of applying these methodologies for capital allocation and planning purposes. Therefore we strongly recommend that the TBG analyses the problem in more detail considering the significance of the calculation in the overall design of the SBA.

The industry is happy to engage into constructive dialogue with the TBG on this problem and discuss the potential solutions described in this initial paper.

APPENDIX 1: Example of Negative Variance in the Risk Aggregation Formula

Suppose we are calculating the Delta Risk Charge for GIRR (General Interest Rate Risk), and that there are just two currencies, with 10 tenor points in each.

Suppose we have WS_1 = 100 and WS_2 = -100 for all tenors, suppose $\rho_{ij} = 40\%$ for all tenors i and j, and suppose $\gamma_{12} = 50\%$. Then we get $K_1 = K_2 = 678$, $S_1 = 1000$, $S_2 = -1000$, and the term inside the square root is -80,000.

Digging deeper into this example, we can show that by expanding the terms into the lowest risk factor level, the implied correlation matrix at the risk factor level is not positive semi-definite. In other words, one is not entirely free in specifying correlation parameters in a cascading approach.

To see this, note that the formula for the delta risk charge is:

$$[\text{Delta Risk Charge}] = \sqrt{\sum_b K_b^2 + \sum_b \sum_{b \neq c} \gamma_{bc} S_b S_c}$$

where

- $K_b = \sqrt{\sum_k WS_k^2 + \sum_k \sum_{l \neq k} \rho_{kl} WS_k WS_l}$
- $S_b = \sum_k WS_k$

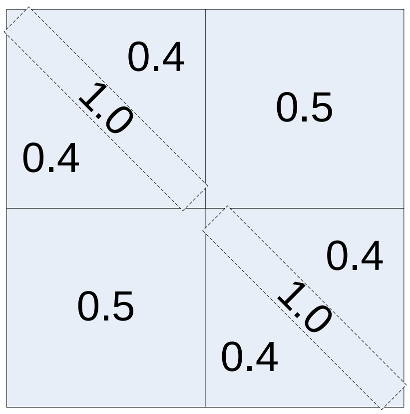
Expanding the charge formula, we have

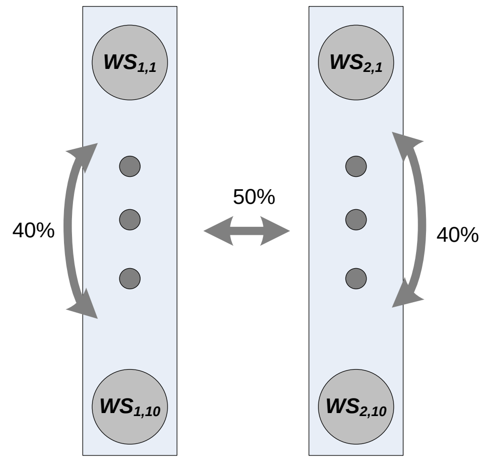
$$[\text{Delta Risk Charge}] = \sqrt{\sum_b \left[\sum_k WS_{b,k}^2 + \sum_k \sum_{l \neq k} \rho_{kl} WS_{b,k} WS_{b,l} \right] + \sum_b \sum_{b \neq c} \sum_k \sum_l \gamma_{bc} WS_{b,k} WS_{c,k}}$$

i.e., the total variance across all delta's based on the following correlations:

- ρ_{kl}: correlation between two points within the same bucket
- γ_{bc}: correlation between two points from different buckets

Taking the example above, we have the following 20x20 '*correlation*' matrix ***C***:





This matrix has a negative eigenvalue (λ=-0.4) where the *WS*'s (i.e. v = [+100,...,+100,-100,..., -100]) in the example is an eigenvector. The *variance* corresponding to this eigenvector is

$$v^T C v = v^T \lambda v = -80{,}000,$$

recovering the result in the example.

APPENDIX 2 : Cascading in SBA - Three Possible Solutions

The SBA methodology has taken a cascading approach to variance calculation.

The main advantages of this approach are:

- Avoids the need to specify a large covariance matrix, with uncertain minor cross-terms
- Easier to specify smaller covariance matrices that are positive semi-definite
- Relatively easy to give explanations of variance back to individual risk causes – helps risks managers and trading staff to understand the drivers of capital.

For a cascading approach to capture hedging benefits across buckets, using K which is like a bucket level standard deviation alone is not sufficient. FRTB2 therefore employed a signed version of bucket risk (S) to capture the hedging effect. Below, we show that beside the particular choice in FRTB2, there are alternative definitions of S that are not exposed to the same negative variance problem.

Three possibilities for the definition of S

Suppose we have three tiers of risk factors (*i*, *j*, *k*), with individual risk weights W_{ijk}. For example the yield curve risk tiers might be (currency, tenor, sub curve), eg (USD, 5y, OIS).

We have three alternative definitions for the signed risk S_{ij}, given the risk capital charge K_{ij},

- 1(a), "Current" $S_{ij} = \sum_k W_{ijk}$
- 2(a), "Signed-K", $S_{ij} = K_{ij} \text{sign}\left(\sum_k W_{ijk}\right)$
- 1(b), "Cap+Floor", $S_{ij} = \min\left(\max\left(\sum_k W_{ijk}, -K_{ij}\right), +K_{ij}\right)$

Cascading equations in full

At the first tier we define the risk capital and its signed version as

$$K_{ij} = \left(\sum_k W_{ijk}^2 + \sum_{k \neq k'} \rho_{kk'} W_{ijk} W_{ijk'}\right)^{1/2},$$

$$S_{ij} = \begin{cases} \sum_k W_{ijk} \\ K_{ij} sign\left(\sum_k W_{ijk}\right) \\ min\left(max\left(\sum_k W_{ijk}, -K_{ij}\right), +K_{ij}\right) \end{cases}$$

Here we have shown three possible alternative definitions for *S*.

Then at the second tier, we define the risk capital as

$$K_i = \left(\sum_j K_{ij}^2 + \sum_{j \neq j'} \rho_{jj'} S_{ij} S_{ij'}\right)^{1/2}.$$

Under option 1(a), the number under the square root can become negative. In this case K_i would be defined as zero. The signed version of the risk capital is defined as

$$S_i = \begin{cases} \sum_j S_{ij} \\ K_i sign\left(\sum_j S_{ij}\right) \\ min\left(max\left(\sum_j S_{ij}, -K_i\right), +K_i\right) \end{cases}$$

Finally, at the last tier, the risk capital is

$$K = \left(\sum_i K_i^2 + \sum_{i \neq i\prime} \rho_{ii\prime} S_i S_{i\prime}\right)^{1/2},$$

and again floor the variance at zero (only if we are using option 1(a)). There is no need to define the signed version now, since this is the final result.

This tiered approach can be easily extended to any level of tiering as required.

Three** S **definitions compared

Option 1(a), "Current", with variance floored at zero

This feels like ignoring the problem rather than solving it. Encountering the square root of a negative number in the capital calculation is surely indicative of a fairly serious problem. It is less than credible to have an approach that could output a capital of zero.

Furthermore, the Delta Risk Charge formula above can give rise to an unreasonably large capital requirement in the case where the risks have the same sign so $S_{ij} > K_{ij}$. In this case the formula does not allow a fair diversification benefit.

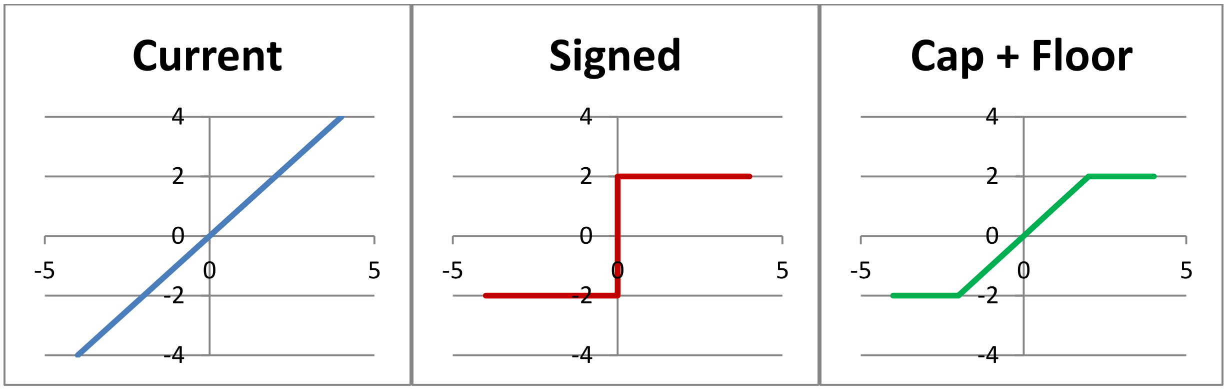
Option 2(a), "Signed-K"

This is a more sophisticated approach which is similar to one used in an earlier version of the SBA. It represents an improvement on Option 1(a), but introduces a new problem, namely that the capital is a discontinuous function of the underlying risks. Since in general $K_{ij} \neq \sum_k WS_{ijk}$, when $\sum_k WS_{ijk}$ goes through zero, then S_{ij} will jump between K_{ij} and $-K_{ij}$. Thus a tiny change in the risks of the portfolio could cause a large change in the capital, which is clearly undesirable.

Option 1(b), "Cap+Floor"

This is similar in spirit to Option 2(a), but avoids the discontinuity problem, since this new definition of S_{ij} is a continuous function of the weighted sensitivities.

We can see the intuitive behaviour of three alternatives as simple graphs. Here the graphs show S against $\Sigma_k W_k$ in the case where $K = 2$.



As can be seen, all of these three methods will work with tiering. Their features can be summarised in this table:

Definition of *S*	**Advantages**	**Disadvantages**
1(a) Current	Simple, continuous	Unbounded, reduces diversification benefit. Possible negative variance, which is cured by flooring the variance at zero.
2(a) Signed-K	Bounded, always non-negative variance.	Discontinuous – capital could move significantly due to tiny change in risk.
1(b) Cap+Floor	Continuous, bounded, always non-negative variance.	More complex formula.

APPENDIX 3: Further Analysis on Possible Solutions (1a) and 1(b)

The purpose of this analysis is an attempt to understand the possible solutions (1.a) and (1.b) in the main text of this document. We keep the same setting as in the example in the appendix, namely,

- two buckets
- ten points per bucket
- ρ_{ij} : 40%

However, we consider two bucket-wise correlations

- γ = 30%: it can be shown that this leads to a legitimate correlation structure.
- γ = 50%: as shown in our previous note, this leads to an illegitimate correlation structure.

We randomly generates the following three different types of the WS_k's, i.e. vectors of 20 elements

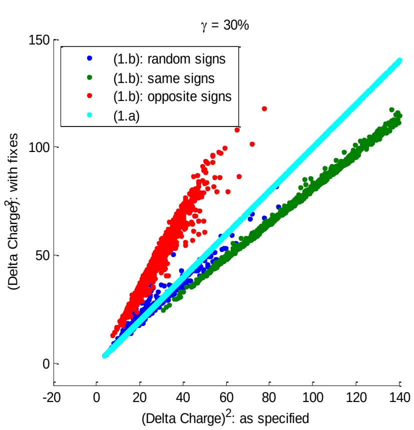
1. random signs: $[x_1, ..., x_{10}, x_{11}, ..., x_{20}]$ where each x_j is sampled using the standard normal.
2. all positive signs: $[|x_1|, ..., |x_{10}|, |x_{11}|, ..., |x_{20}|]$
3. opposite signed between buckets: $[+|x_1|, ..., +|x_{10}|, -|x_{11}|, ..., -|x_{20}|]$

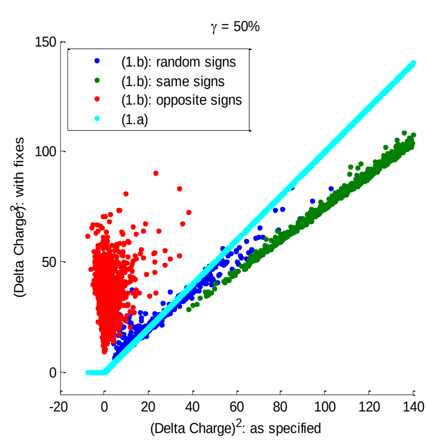
The third type is motivated by the fact that the eigenvector associated with negative eigenvalue with γ = 50% is [+1,..., +1,-1, ... -1].

For each combination, we calculate the square of the delta charges as follows:

- specified in FRTB2
- in the ways specified in the potential solution 1.
  1. (1.a): simple flooring
  2. (1.b): caps/floors

1000 simulations are run.





Observations:

- Solution (1.a): With $\gamma = 50\%$, this approach can generate negative values and should be floored. Otherwise, the results are identical to the standard variances.
- Solution (1.b): Even with $\gamma = 30\%$, where the correlation is legitimate, the approach gives different values from the standard variance. It can yield values systemically higher (opposite sign case) or lower (same sign case) than the standard variances. Focusing on the case of $\gamma = 50\%$ and opposite signs, there seems to be no clear relationship with the standard variances. Therefore, it seems to require further analysis to understand the approach.

APPENDIX 4: Possible Solution (4) Single Correlations Scaled by γ

As discussed in Appendix 1, the root cause of the problem of having a negative variance is a misspecification of the correlation structure.

As having negative eigenvalues can potentially lead to lower capital charges, it is desirable to specify legitimate correlation structures.

- Nonetheless, it may not be an easy task to prove that the corrections are correctly specified, i.e. positive semi-definite.
- Having a single correlation γ_{bc} between two different buckets would not be ideal, e.g. USD 1Y and EUR 1Y would be more correlated than USD 1Y and EUR 30Y.

This solution corrects the fundamental problem that inconsistent correlations can result in negative variance. It is a simple way of specifying the correlations between risks in different currencies/buckets in such a way that global correlation matrix is self-consistent (ie. positive semi-definite).

- No additional inputs to those that are currently used. Correlations are derived using the existing single currency correlation matrix and the γ values specified in the rule.
- Compatible with solution (1.a) as a failsafe in case modifications to the calculation introduce the possibility of negative variance.

For brevity, we refer to risk for a "currency". This can be generalized to "currency", "curve", or "bucket" as appropriate. Similarly, exposures by "maturity" can be generalized to exposures by "product", "credit class", etc.

Re-cap of Current Method: The current method uses the following formulas:

(1) $S_b = \sum_i WS_{b,i}$

(2) $[\text{Delta Risk Charge}] = \sqrt{\sum_b K_b^2 + \sum_b \sum_{c \neq b} \gamma_{bc} S_b S_c}$

For clarity, we define a value X which is the cross variance between 2 currencies. Using X in the current method, we get

(3) $X_{b,c} = S_b S_c$ Note that this is equivalent to $X_{b,c} = \sum_i \sum_j WS_{b,i} WS_{c,j}$

(4) $[\text{Delta Risk Charge}] = \sqrt{\sum_b K_b^2 + \sum_b \sum_{c \neq b} \gamma_{bc} X_{bc}}$

Solution 4a: We propose changing the formula for X to use the single currency correlations in the calculation.

(3a) $X_{b,c} = \sum_i \sum_j \rho_{ij} WS_{b,i} WS_{c,j}$

Then use (4) to calculate the delta risk charge.

Economically, this has a simple interpretation. The correlations between currencies are simply a scaled down version of the correlations within a currency. γ is the scale factor and can range from 0 to 1. For example, suppose $\gamma = 0.5$ and we have 2 exposures in the 10y bucket. If they are the same currency, the correlation would be 1.0. If they are in different currencies, the correlation is 0.5 * 1.0 = 0.5. If we had same sign exposures in the 5y and 10y bucket, the single currency correlation is 0.9. For exposures in different currencies the correlation would be 0.45.

On Validity of Proposal: If the single currency correlation matrix is valid, the proposed method always results in a non-negative variance: Let C be a valid correlation matrix for a single currency. x and y will be vectors for risk weighted exposures.

(a) C is positive semi-definite, so for all vectors of real numbers x: $x^T C x \geq 0$

Let x and y represent portfolios in 2 different currencies. The variance of x+y will be

(b) $(x+y)^T C(x+y) \geq 0$

The variance can be rewritten as follows

$$
\begin{aligned}
&(x+y)^T C(x+y) \\
&\quad = x^T Cx + x^T Cy + y^T Cx + y^T Cy \\
&\quad = x^T Cx + y^T Cy + 2 * y^T Cx
\end{aligned}
$$

Putting the new form into (b) we get

(c) $x^T Cx + y^T Cy + 2 * y^T Cx \geq 0$

The 3 terms of the variance of x+y are the variance of x, the variance of y, and the contribution of the cross correlation between x and y. The first 2 terms are the K_b terms in (4). We know that this quantity is non-negative.

$$x^T Cx + y^T Cy = \sum_b K_b^2$$

The third term is the cross currency exposure and corresponds to the second part in (4), but without the scale factor.

$$2 * y^T Cx = \sum_b \sum_{c \neq b} X_{bc}$$

Since the first 2 terms are non-negative and the total is non-negative, multiplying the third term by a constant, γ, between 0 and 1 must result in a non-negative sum.

$$x^T Cx + y^T Cy + 2\gamma_{bc} * y^T Cx = \sum_b K_b^2 + \sum_b \sum_{c \neq b} \gamma_{bc} X_{bc} \geq 0$$

If γ is the same for all currency pairs, which is the current rule, the result applies to any number of currencies. We recommend that the committee keep this specification.

Notes:

- If the committee chooses to specify different γ for different currency pairs we cannot rule out the possibility of negative variance. In particular, it is possible to specify inconsistent values for triplets of currencies. The only solution would be a validation of the full correlation matrix.
- Taking into account the same- and different-sign correlations, the correlation structures $C^+ = c^+_{ij}$ and $C^- = c^-_{ij}$ over all maturities across the buckets would look like the followings:

ρ^+_{ij}	$\gamma\,\rho^+_{ij}$	$\ddots$	$\gamma\,\rho^+_{ij}$
$\gamma\,\rho^+_{ij}$	ρ^+_{ij}	$\ddots$	$\ddots$
$\ddots$	$\ddots$	$\ddots$	$\gamma\,\rho^+_{ij}$
$\gamma\,\rho^+_{ij}$	$\ddots$	$\gamma\,\rho^+_{ij}$	ρ^+_{ij}

ρ^-_{ij}	$\gamma\,\rho^-_{ij}$	$\ddots$	$\gamma\,\rho^-_{ij}$
$\gamma\,\rho^+_{ij}$	ρ^-_{ij}	$\ddots$	$\ddots$
$\ddots$	$\ddots$	$\ddots$	$\gamma\,\rho^-_{ij}$
$\gamma\,\rho^-_{ij}$	$\ddots$	$\gamma\,\rho^-_{ij}$	ρ^-_{ij}

Then, c^+_{ij} and c^-_{ij} satisfy the conditions in Appendix 5 if

- $\rho^+_{ij} \geq \rho^-_{ij} \geq 0$
- One of ρ^+_{ij} and ρ^-_{ij} is positive definite.

- On Cascading:
  - In this approach, the strict bucket-by-bucket 'cascading' is lost: (3a) should be calculated across all pairs.
  - On the other hand, the delta charge is an aggregated sum of the following extended cross-cascading structure K_b's and X_{bc}'s:

Kb	Value
GBP	34
EUR	13
USD	43
JPY	22

Xbc	GBP	EUR	USD	JPY
GBP		4	2	3
EUR			-3	-2
USD				1
JPY				

- In case of three or more cascading, the approach can be rather complex than other cascading approaches in Appendix 4.

APPENDIX 5: Same and Different Sign Correlations

The purpose of this section is to present a condition where the same- and different sign correlations to satisfy in order to ensure the positive definiteness.

Let c_{ij}^{+} and c_{ij}^{-} be the correlations for the pairs of the same and different signs, respectively.

Assume that

- $c_{ij}^{+} \geq c_{ij}^{-} \geq 0$
- One of c_{ij}^{+} and c_{ij}^{-} is positive definite.

Then, we have

$$v^{t}Cv = \sum_{v_i v_i \geq 0} c_{ij}^{+} v_i v_j + \sum_{v_i v_i < 0} c_{ij}^{-} v_i v_j \geq \sum c_{ij}^{-} v_i v_j \geq 0$$

if c_{ij}^{-} is positive definite. A similar proof can be made if c_{ij}^{+} is positive definite.