Guidance Note – Linear Interpolation

Introduction

ISDA published a Best Practice Statement for Linear Interpolations on 18th December 2009. More recently, members of the ISDA Rates Market Infrastructure Working Group noted that such document did not in certain circumstances clarify how to determine the number of calendar days corresponding to the next shorter and next longer Designated Maturities of the two known rates in certain situations, including, e.g., when ‘end-of-month’ convention applies.

In addition to the above, the December 2009 Best Practice Statement did not cover interpolation under the ISDA 2013 Discontinued Rates Maturities Protocol, the IBOR Fallbacks Supplement (Supplement 70 to the 2006 ISDA Definitions) and the ISDA 2020 IBOR Fallbacks Protocol, the 2021 IBOR Fallbacks Supplement (Supplement 90 to the 2006 ISDA Definitions) and the December 2021 Benchmark Module to the ISDA 2021 Fallbacks Protocol or the 2021 ISDA Interest Rate Derivatives Definitions.

This Guidance Note therefore seeks to provide further clarification for linear interpolations under these documents.

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Footnote:

1 This paper does not constitute legal or accounting advice and members should seek advice from their own professional advisers in respect of the terms of the 2006 ISDA Definitions, the 2021 ISDA Interest Rate Derivatives Definitions, the IBOR Fallbacks Supplement, the ISDA 2020 IBOR Fallbacks Protocol, the 2021 IBOR Fallbacks Supplement, the December 2021 Benchmark Module to the ISDA 2021 Fallbacks Protocol and the ISDA 2013 Discontinued Rates Maturities Protocol outlined in this document.
**Linear Interpolation**

“Linear Interpolation” is generally specified to be applicable for Calculation Periods (or Compounding Periods), where there is no Designated Maturity of the relevant Floating Rate Option that corresponds to the length of that particular Calculation Period (or Compounding Period).

Linear Interpolation assumes that the unknown rate \( R_n \) lies on the line (AC) between the two known rates. Because AC is linear, that is, a straight line, the slope of the line (AB) connecting \( R_1 \) and \( R_n \) is the same as the slope of line (AC) connecting \( R_1 \) and \( R_2 \). Using the “rise over run” formula for the slope of the line, we solve for \( R_n \) as follows:

\[
R_n = R_1 + \frac{R_2 - R_1}{t_2 - t_1} x (t_n - t_1)
\]

Figure 1: Linear interpolation
2006 ISDA Definitions – Section 8.3. Interpolation

Under Section 8.3 of the 2006 ISDA Definitions, the Linear Interpolation calculation is described in a paragraph of narrative text. It requires that the rate is determined through the use of “straight-line interpolation” by reference to two rates, one of which is determined as if the Designated Maturity were the period of time for which rates based on the relevant Floating Rate Option are available next shorter than the length of the relevant Calculation Period (or Compounding Period) and the other of which is determined as if the Designated Maturity were the period of time for which rates based on the relevant Floating Rate Option are available next longer than the length of the relevant Calculation Period (or Compounding Period) (or, in each case, any alternative Designated Maturity agreed to by the parties).

The definition of Linear Interpolation in the 2006 ISDA Definitions (Section 8.3) does not, however, express how to determine the number of days in the periods corresponding to the next longer and next shorter Designated Maturities of the two known rates that are required under a straight-line interpolation calculation ($t_1$ and $t_2$ in the formula on page 2 above); in particular, when one or both of the corresponding period end dates are not good business days, it is not specified which business day calendar/convention (if any) should be used to adjust the corresponding dates in order to determine number of days in such periods.

ISDA understands that market practice regarding the determination of the start and end dates of the longer and shorter periods (and therefore the number of days in such periods) for Linear Interpolations under the 2006 ISDA Definitions, is to use the same conventions and adjustments (e.g. the same Business Days and Business Day Convention) that apply to the determination of the Period End Dates (or Compounding Dates) under the relevant Swap Transaction (i.e. determined and adjusted as if such periods are Calculation Periods (or Compounding Periods) under the relevant Swap Transaction).

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2 Capitalized terms used and not otherwise defined in this section on the 2006 ISDA Definitions have the meaning given to them in the 2006 ISDA Definitions.
Under Section 6.10 of the 2021 ISDA Interest Rate Derivatives Definitions, the Linear Interpolation calculation is set out as a precise formula, representing the market practice described above for Linear Interpolation calculations under the 2006 ISDA Definitions.

The formula is set out as follows:

\[ R_n = R_1 + \frac{R_2 - R_1}{t_2 - t_1} \times (t_n - t_1) \]

where:

(i) “\(R_1\)” means the rate for the Reset Date determined from the Floating Rate Option as if the Designated Maturity were the Shorter Designated Maturity;

(ii) “**Shorter Designated Maturity**” means the period of time for which rates are available and which are not Non-Representative (if Non-Representative applies in respect of the Floating Rate Option) that is next shorter than the Calculation Period or Compounding Period to which the Reset Date relates (or any alternative shorter Designated Maturity agreed by the parties);

(iii) “\(R_2\)” means the rate for the Reset Date determined from the Floating Rate Option as if the Designated Maturity were the Longer Designated Maturity;

(iv) “**Longer Designated Maturity**” means the period of time for which rates are available and which are not Non-Representative (if Non-Representative applies in respect of the Floating Rate Option) that is next longer than the Calculation Period or Compounding Period to which the Reset Date relates (or any alternative longer Designated Maturity agreed by the parties);

(v) “\(t_1\)” means the number of calendar days from and including S to but excluding \(P_1\);

(vi) “\(t_2\)” means the number of calendar days from and including S to but excluding \(P_2\);

(vii) “\(P_1\)” means, if the Shorter Designated Maturity is: (a) one day, the calendar day immediately following S, subject to adjustment in accordance with the Following Business Day Convention; (b) one week, two weeks or three weeks, the calendar day that is the corresponding number of weeks immediately following S, subject to adjustment in accordance with the Business Day Convention.

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3 Capitalized terms used and not otherwise defined in this section on the 2021 ISDA Interest Rate Derivatives Definitions have the meaning given to them in the 2021 ISDA Interest Rate Derivatives Definitions.
Convention applicable to Period End Dates or Compounding Dates (as applicable); or (c) a period of months or years, the calendar day that is the corresponding number of months or years, as applicable, immediately following S (but if S is the 29th, 30th or the 31st day of a month and there is no corresponding numbered day in the month that is the relevant number of months following S, the last calendar day in the month that is the relevant number of months following S), subject to adjustment in accordance with the Business Day Convention applicable to Period End Dates or Compounding Dates (as applicable);

(viii) “P2” means, if the Longer Designated Maturity is: (a) one week, two weeks or three weeks, the calendar day that is the corresponding number of weeks immediately following S, subject to adjustment in accordance with the Business Day Convention applicable to Period End Dates or Compounding Dates (as applicable); or (b) a period of months or years, the calendar day that is the corresponding number of months or years, as applicable, immediately following S (but if S is the 29th, 30th or 31st day of a month and there is no corresponding numbered day in the month that is the relevant number of months following S, the last calendar day in the month that is the relevant number of months following S), subject to adjustment in accordance with the Business Day Convention applicable to Period End Dates or Compounding Dates (as applicable);

(ix) “S” means the first day of the Calculation Period or Compounding Period (as applicable); and

(x) “tn” means the number of calendar days in the Calculation Period or Compounding Period (as applicable).
Interpolation under the ISDA 2013 Discontinued Rates Maturities Protocol

The terms of the ISDA 2013 Discontinued Rates Maturities Protocol (the DRM Protocol) apply to a Protocol Covered Transaction if both parties to the trade have adhered to the DRM Protocol.

The ISDA 2013 Discontinued Rates Maturities Protocol provides that linear interpolation will apply if any rate is to be determined by reference to one or more Designated Maturities that is or has been permanently discontinued (as announced by the Provider thereof or an agent of the Provider) and provided that both a “Nearest Long Rate” and a “Nearest Short Rate” are available. In that instance, the rate for the Reset Date will be the “Interpolated Rate”, which is determined by using linear interpolation (unless another method of interpolation is specified in the Confirmation) between the next shorter and next longer tenors that continue to be published.5

In relation to a rate which is originally determined by reference to the interpolation of two specified rates, one of which is then discontinued (an Affected Interpolated Rate), the DRM Protocol provides that the rates which are next longer and next shorter than the number of days during the relevant Calculation Period or Compounding Period (the Affected Interpolated Rate Period) will be interpolated to determine the rate. For example, if the Calculation Period was 45 calendar days and the Confirmation stated that the rate for such Calculation Period was calculated by Linear Interpolation between the 1 month and 2 month rate, and 2 month rate is discontinued, the DRM Protocol provides that you interpolate between the next shorter and longer rates determined by reference to the length of the Calculation Period (or Compounding Period). For a 45 day Calculation Period, this means you would interpolate between the 1 month rate and the 3 month rate (provided such Designated Maturities continue to be published) to determine the rate for the Reset Date.

Note that the DRM Protocol only applies to Protocol Covered Transactions that relate to one or more discontinued Designated Maturities where relevant longer and shorter tenors continue to be published.

The material terms of the DRM Protocol as amended by the IBOR Fallbacks Supplement and the ISDA 2020 IBOR Fallbacks Protocol have been incorporated into the 2021 ISDA Interest Rate Derivatives Definitions and therefore apply to trades confirmed under that definitional booklet regardless of whether both parties have adhered to the DRM Protocol. The relevant provisions have been significantly simplified compared to the DRM Protocol but no substantive changes have been made except for a clarification to what happens when no shorter or longer tenor is available – in this case, the fallbacks that operate upon a permanent cessation will apply.

4 Capitalized terms used and not otherwise defined in this section on the ISDA 2013 Discontinued Rates Maturities Protocol have the meaning given to them in the ISDA 2013 Discontinued Rates Maturities Protocol.
Interpolation under the IBOR Fallbacks Supplement, the ISDA 2020 IBOR Fallbacks Protocol, the 2021 IBOR Fallbacks Supplement and the December 2021 Benchmark Module to the ISDA 2021 Fallbacks Protocol

Linear interpolation is relevant in two instances under the IBOR Fallbacks Supplement and the ISDA 2020 IBOR Fallbacks Protocol as well as under the 2021 IBOR Fallbacks Supplement and the December 2021 Benchmark Module to the ISDA 2021 Fallbacks Protocol.

Firstly, similar to the DRM Protocol, linear interpolation will apply if certain but not all tenors of an IBOR are permanently discontinued (or, in the case of LIBOR, Non-Representative) and the discontinued rates maturities provisions in Section 8.5 of the 2006 ISDA Definitions (as amended by the IBOR Fallbacks Supplement and the 2021 IBOR Fallbacks Supplement) apply. In that instance and assuming there is both a “Nearest Long Rate” and a “Nearest Short Rate”, the rate for the permanently discontinued (or Non-Representative, as applicable) IBOR tenor is determined by interpolating the next shorter and next longer tenors that continue to be published (and, in the case of LIBOR, are not Non-Representative). If either a Nearest Long Rate or a Nearest Short Rate does not continue to be published (or, in the case of LIBOR, is Non-Representative) then the fallbacks that operate upon a permanent cessation will apply. Note that if both a Nearest Long Rate and a Nearest Short Rate exist relative to the tenor that has been permanently discontinued (i.e. have not been permanently discontinued (or, are, are not Non-Representative, as applicable)), but the rate for one or both of the associated Designated Maturities of the Nearest Long Rate and a Nearest Short Rate does not appear on the relevant page or is not published by the Administrator as of the relevant time on the relevant day (as per the relevant Floating Rate Option), then linear interpolation will continue to apply to the tenor that is permanently discontinued (or Non-Representative, as applicable), but such Nearest Long Rate and/or Nearest Short Rate, as the case may me, will be determined in accordance with the fallbacks that apply under the relevant Floating Rate Option where no Index Cessation Effective Date has occurred (the Temporary Non-Publication Fallback under the 2021 ISDA Interest Rate Derivatives Definitions).

Secondly, in respect of a Calculation Period to which “Linear Interpolation” is specified to be applicable (pursuant to Section 8.3 of the 2006 ISDA Definitions), Section 7.9 of the 2006 ISDA Definitions (as amended by the IBOR Fallbacks Supplement and the 2021

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6 Capitalized terms used and not otherwise defined in this section have the meanings given to them in the IBOR Fallbacks Supplement and the ISDA 2020 IBOR Fallbacks Protocol or the 2021 IBOR Fallbacks Supplement (Supplement 90 to the 2006 ISDA Definitions) and the December 2021 Benchmark Module to the ISDA 2021 Fallbacks Protocol, as applicable. Note also that the substance of all of these provisions have been incorporated into the 2021 ISDA Interest Rate Derivatives Definitions.

7 The discontinued rates maturities provisions in Section 8.5 of the 2006 ISDA Definitions, as amended by the IBOR Fallbacks Supplement and the 2021 IBOR Fallbacks Supplement, are based on the operative provisions in the ISDA 2013 Discontinued Rates Maturities Protocol. However, there are some differences (for example, the provisions in Section 8.5 of the 2006 ISDA Definitions may apply if a tenor of LIBOR is discontinued or Non-Representative whereas the ISDA 2013 Discontinued Rates Maturities Protocol only applies if a tenor is a discontinued rate).
IBOR Fallbacks Supplement) provides that, if there is no longer and/or no shorter rate which can be used to determine the rate for the Reset Date, then the Adjusted RFR (i.e. the relevant risk-free rate compounded in arrears over the offset period) plus the Interpolated Spread shall apply. If a “Spread Adjustment Fixing Date” has occurred for both tenors that are shorter and longer than the length of the relevant Calculation Period or Compounding Period, then the fixed spread for the next shorter and next longer tenors will be interpolated to determine the Interpolated Spread.\(^8\) Note that in the calculation of the Interpolated Spread, no offsets are applied to the determination of the number of calendar days in the periods corresponding to the next shortest and next longest Tenors.

Note that Section 7.9 of the 2006 ISDA Definitions (as amended by the IBOR Fallbacks Supplement and the 2021 IBOR Fallbacks Supplement) only applies to certain Sterling LIBOR, Swiss Franc LIBOR, U.S. Dollar LIBOR, Euro LIBOR, EURIBOR, Yen LIBOR, Yen TIBOR, Euroyen TIBOR, BBSW, CDOR, HIBOR, STIBOR, SIOR, NIBOR, BKBM and KLIBOR Floating Rate Options. It does not apply to SOR and THBFIX Floating Rate Options, as amended by the IBOR Fallbacks Supplement, or the PHIREF and MIFOR Floating Rate Options, as amended by the 2021 IBOR Fallbacks Supplement.

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\(^8\) If a “Spread Adjustment Fixing Date” has not occurred for both tenors that are shorter and longer than the length of the relevant Calculation Period or Compounding Period, then the spread is equal to the spread for the nearest tenor (either shorter or longer) for which a “Spread Adjustment Fixing Date” has occurred.
**Rounding**

As per Section 8.1 of the 2006 ISDA Definitions and Section 4.8 of the 2021 ISDA Interest Rate Derivatives Definitions, unless otherwise specified in the relevant Confirmation or, in the case of an Average Floating Rate Option, a Compounded Floating Rate Option, an Overnight Floating Rate Option used in conjunction with an Overnight Rate Compounding Method or an Overnight Rate Averaging Method, or a Compounded Index Floating Rate Option under the 2021 ISDA Interest Rate Derivatives Definitions, unless otherwise specified in the Compounding/Averaging Matrix, all percentages resulting from calculations under those definitional booklets are rounded to the nearest one hundred-thousandth of a percentage point (0.00001%), except that if the percentage is determined through the use of interpolation by reference to two rates based on a Floating Rate Option, then the resulting percentage is rounded to the same degree of accuracy as those two rates (but not to a lower degree of accuracy than the nearest one thousandth of a percentage point (0.001%)).

Note that under these rounding provisions, ties are rounded away from zero and only the resulting percentage is rounded, not intermediate calculations.
Example – Linear Interpolation under the 2021 ISDA Interest Rate Derivatives Definitions (note that this is also market practice for Linear Interpolation calculations under the 2006 ISDA Definitions)

In this example, a hypothetical transaction with the following terms has a short initial stub period:

Trade Date: 06 January 2022
Effective Date: 10 January 2022
Business Days: London and New York
Business Day Convention: Modified Following
Floating Rate Option: USD-LIBOR
Designated Maturity: 3 Months
Linear Interpolation: Applicable

Initial Calculation Period: The first day of the initial Calculation Period (the Effective Date) is 10 January 2022 (subject to No Adjustment Business Day Convention), the last day of the initial Calculation Period (the first Period End Date) is 21 March 2022 (the 20 March 2022 is not a Business Day and is adjusted in accordance with the Modified Following Business Day Convention). The number of calendar days in the initial Calculation Period (t₀) is therefore 70.

Linear Interpolation calculation:

Shorter Designated Maturity = 1 Month (the next shorter available Designated Maturity).

Longer Designated Maturity = 3 Months (the next longer available Designated Maturity).

R₁ = 0.10414% (the rate determined as if the Designated Maturity were the Shorter Designated Maturity).

R₂ = 0.23129% (the rate determined as if the Designated Maturity were the Longer Designated Maturity).

S = 10 January 2022 (the first day of the Calculation Period).

P₁ = 10 February 2022 (the calendar day that is 1 month immediately following S, subject to adjustment in accordance with the Modified Following Business Day Convention).

P₂ = 11 April 2022 (the calendar day that is 3 months immediately following S, subject to adjustment in accordance with the Modified Following Business Day Convention).

t₁ = 31 days (the number of calendar days from and including S to but excluding P₁).
$t_2 = 91$ days (the number of calendar days from and including $S$ to but excluding $P_2$).

We solve for $R_n$ as follows:

$$R_n = R_1 + \frac{R_2 - R_1}{t_2 - t_1} \times (t_n - t_1)$$

$$= 0.10414\% + \frac{0.23129\% - 0.10414\%}{91 - 31} \times (70 - 31)$$

$$= 0.10414\% + \frac{0.12715\%}{60} \times 39$$

$$= 0.10414\% + 0.002119\% \times 39$$

$$= 0.18679\%$$

Therefore, the Relevant Rate in respect of the initial Calculation Period is 0.18679\% (rounded to 5 d.p.)