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Sent by email to: baselcommittee@bis.org

Consultative Document: Fundamental review of the trading book[1]- further response

Dear Ms. Barger and Mr. Adkins,

This letter contains a further response of the International Swaps and Derivatives Association, Inc[2] ("ISDA"), the Global Financial Markets Association[3] ("GFMA") and the Institute of International Finance[4] (together "the Associations"), to the Basel Committee on Banking Supervision ("BCBS") Consultative Document *Fundamental Review of the Trading Book* dated May 2012 ("Fundamental Review" or "FRTB"). This paper should be read in the context of the previous industry response submitted in September 2012 and the paper Diversification and Model Approval submitted in February 2013.

The Associations very much appreciate the opportunity to comment further on the Fundamental Review and to meet with the BCBS Trading Book Group ("TBG") in Washington in June 2012 and in Frankfurt in August 2012 and in other bilateral meetings. We found those meetings to be constructive

[1] Basel Committee on Banking Supervision, May 2012

[2] Since 1985, ISDA has worked to make the global over-the-counter (OTC) derivatives markets safer and more efficient. Today, ISDA is one of the world's largest global financial trade associations, with over 840 member institutions from 59 countries on six continents. These members include a broad range of OTC derivatives market participants: global, international and regional banks, asset managers, energy and commodities firms, government and supranational entities, insurers and diversified financial institutions, corporations, law firms, exchanges, clearinghouses and other service providers. Information about ISDA and its activities is available on the Association's web site: www.isda.org.

[3] The Global Financial Markets Association (GFMA) brings together three of the world's leading financial trade associations to address the increasingly important global regulatory agenda and to promote coordinated advocacy efforts. The Association for Financial Markets in Europe (AFME) in London and Brussels, the Asia Securities Industry & Financial Markets Association (ASIFMA) in Hong Kong and the Securities Industry and Financial Markets Association (SIFMA) in New York and Washington are, respectively, the European, Asian and North American members of GFMA. For more information, please visit http://www.gfma.org .

[4] The Institute of International Finance, Inc. (IIF) is a global association created in 1983 in response to the international debt crisis. The IIF has evolved to meet the changing needs of the international financial community. The IIF's purpose is to support the financial industry in prudently managing risks, including sovereign risk; in disseminating sound practices and standards; and in advocating regulatory, financial, and economic policies in the broad interest of members and foster global financial stability. Members include the world's largest commercial banks and investment banks, as well as a growing number of insurance companies and investment management firms. Among the IIF's Associate members are multinational corporations, consultancies and law firms, trading companies, export credit agencies, and multilateral agencies. All of the major markets are represented and participation from the leading financial institutions in emerging market countries is also increasing steadily. Today the IIF has more than 450 members headquartered in more than 70 countries.

and assisted the industry to formulate its responses in a focused way. This paper follows the earlier paper on Diversification and Model Approval. It addresses the calibration of the two parameters presented in the previous paper; Alpha which governs diversification benefit and Beta which controls the penalty for poor model performance. We also submit a short paper on the use of the Kalman Filter.

We stress again that we are very broadly in agreement with the direction of the FRTB and feel that the points set out in this paper complement the FRTB and better calibrate the proposals. We would welcome the opportunity to discuss with you further the issues set out in this paper and value any other areas where you feel our input would be helpful.

Yours faithfully,

Peter Sime	Simon Lewis	Andrés Portilla
Head of Risk and Capital	CEO	Director of Regulatory Affairs
ISDA	GFMA	IFF

cc: Wayne Byres, Secretary General, Basel Committee on Banking Supervision

Document classification: **Public**

Dynamic Linear Models - Kalman Filter and Exponential Smoothing

version 1.0

David Phillips
Royal Bank of Scotland

March 25, 2013

Executive Summary

This short note recaps on the Kalman Filter recursive update equations for dynamic linear models, and shows the conditions under which the optimal forecast is equivalent to exponential smoothing.

Contents

1 Update log

11-Mar-2013 Document created.

25-Mar-2013 Minor format changes.

2 Introduction

As part of Joint Industry response to the May 2012 Basel Committee on Banking Supervision (BCBS) consultative document 'Fundamental Review of the Trading Book', an alternative framework is proposed for quantifying the degree of diversification benefit to be allowed in internal models as well as the transition to standard rules in the event of poor model performance.

The purpose of this note is to provide further proof of the linkage between the Kalman Filter recursive algorithms for forecasting in the case of dynamic linear models, and the familiar exponential smoothing approach. This linkage is referenced in one of the proposals in the Joint Industry paper for estimation of α and β.

3 Model setup

We will assume the following dynamic linear model equations:

$$y_t = {F_t}^T \cdot \theta_t + \nu_t \tag{1}$$

and

$$\theta_t = G_t \cdot \theta_{t-1} + \omega_t \tag{2}$$

The first equation, the *Observation Equation*, relates the observation y_t at time t to an unknown state variable θ_t. The second equation, the *State Equation*, describes how the state variable θ_t evolves through time.

We further assume that $\nu_t \sim N(0, V_t)$, $\omega_t \sim N(0, W_t)$, and $\theta_0 \sim N(m_0, C_0)$. The error terms ν_t and ω_t are independent of each other and serially uncorrelated (i.e. ν_t is not correlated with ν_{t-1}).

4 Kalman Filter update equations

The standard model setup introduced above leads to a set of recursive equations for forecasting θ_t, the so-called Kalman Filter (see, for example, Harrison and Stevens 1976).

For the prior distribution of θ_t given all the information up to and including time $t-1$ (denoted by D_{t-1}):

$$\theta_t | D_{t-1} \sim N(a_t, R_t), \tag{3}$$

where

$$\begin{aligned} a_t &= G_t \cdot m_{t-1} \\ R_t &= G_t \cdot C_{t-1} \cdot {G_t}^T + W_t \end{aligned}$$

For the posterior distribution of θ_t additionally conditioned on the observation y_t, or equivalently $D_t \equiv D_{t-1} \cup y_t$:

$$\theta_t | D_t \sim N(m_t, C_t), \tag{4}$$

where

$$\begin{aligned} m_t &= a_t + R_t \cdot F_t \cdot {Q_t}^{-1} \cdot (y_t - {F_t}^T \cdot a_t) \\ C_t &= R_t - R_t \cdot {Q_t}^{-1} \cdot {R_t}^T \\ Q_t &= {F_t}^T \cdot R_t \cdot F_t + V_t \end{aligned}$$

5 Simplified case

For our purposes we are mainly interested in a simplied version of the above setup, where $F_t = 1$, $G_t = 1$, $V_t = V$, and $W_t = W$ for all t. In this case, we can see that the state variable θ_t evolves as a simple random walk, and that the observations y_t are centred around θ_t.

This leads to the following simplifications of the parameters in equations 3 and 4:

$$\begin{aligned} a_t &= m_{t-1} \\ R_t &= C_{t-1} + W \end{aligned}$$

and

$$\begin{aligned} m_t &= a_t + R_t \cdot {Q_t}^{-1} \cdot (y_t - a_t) \\ C_t &= R_t - R_t \cdot {Q_t}^{-1} \cdot {R_t}^T \\ Q_t &= R_t + V \end{aligned}$$

We first consider the recursive equations for the update of the mean m_t.

$$\begin{aligned} m_t &= a_t + R_t \cdot {Q_t}^{-1} \cdot (y_t - a_t) \\ &= m_{t-1} + (C_{t-1} + W) \cdot (C_{t-1} + W + V)^{-1} \cdot (y_t - m_{t-1}) \\ &= m_{t-1} + \frac{C_{t-1} + W}{C_{t-1} + W + V} \cdot (y_t - m_{t-1}) \\ &= \left(1 - \frac{C_{t-1} + W}{C_{t-1} + W + V}\right) \cdot m_{t-1} + \frac{C_{t-1} + W}{C_{t-1} + W + V} \cdot y_t \\ &= (1 - \lambda_{t-1}) \cdot m_{t-1} + \lambda_{t-1} \cdot y_t \end{aligned}$$

where

$$\lambda_{t-1} \equiv \frac{C_{t-1} + W}{C_{t-1} + W + V} \tag{5}$$

Thus, it can be seen that the recursive updating of the mean forecast m_t is equivalent to an exponentially smoothed forecast with (time-varying) parameter λ_{t-1}. We will subsequently show the conditions for which this parameter does not depend on t, i.e. $\lambda_{t-1} = \lambda$ for all t.

The equation for the variance C_t can be written:

$$\begin{aligned} C_t &= R_t \cdot \left(1 - \frac{R_t}{R_t + V}\right) \\ &= (C_{t-1} + W) \cdot \left(1 - \frac{C_{t-1} + W}{C_{t-1} + W + V}\right) \end{aligned}$$

In order for λ_{t-1} to not depend on t, we must equivalently have that $C_t = C_{t-1} = C$. We will now explore the conditions under which this statement holds.

$$\begin{aligned} C_t = C_{t-1} \quad &\Leftrightarrow \quad C_{t-1} = (C_{t-1} + W) \cdot \left(1 - \frac{C_{t-1} + W}{C_{t-1} + W + V}\right) \\ &\Leftrightarrow \quad C_{t-1} \cdot (C_{t-1} + W + V) = (C_{t-1} + W) \cdot V \\ &\Leftrightarrow \quad {C_{t-1}}^2 + C_{t-1} \cdot W - W \cdot V = 0 \\ &\Leftrightarrow \quad (C_{t-1} + \frac{1}{2}W)^2 = W \cdot (\frac{1}{4}W + V) \\ &\Leftrightarrow \quad C_{t-1} = -\frac{1}{2}W + \sqrt{W \cdot (\frac{1}{4}W + V)} \end{aligned}$$

Without loss of generality, we can write $V = kW$ for some $k > 0$. Substitution into the above leads to:

$$\begin{aligned} C_t = C_{t-1} \quad &\Leftrightarrow \quad C_{t-1} = -\frac{1}{2}W + \sqrt{W \cdot (\frac{1}{4}W + kW)} \\ &\Leftrightarrow \quad C_{t-1} = -\frac{1}{2}W + W \cdot \sqrt{\frac{1}{4} + k} \\ &\Leftrightarrow \quad C_{t-1} = W \cdot \left(-\frac{1}{2} + \sqrt{\frac{1}{4} + k}\right) \end{aligned}$$

We proceed by assuming the initial variance C_0 can be written in this way, i.e.

$$C_0 = W \cdot \left(-\frac{1}{2} + \sqrt{\frac{1}{4} + k}\right) \tag{6}$$

At time $t = 1$ we have that

$$C_1 = (C_0 + W) \cdot \left(1 - \frac{C_0 + W}{C_0 + W + kW}\right) \tag{7}$$

which after substitution using the previous equation for C_0 gives

$$\begin{aligned} C_1 &= W\cdot\left(\frac{1}{2}+\sqrt{\frac{1}{4}+k}\right)\cdot\frac{kW}{W\cdot\left(\frac{1}{2}+\sqrt{\frac{1}{4}+k}\right)+kW} \\ &= W\cdot\frac{\left(\frac{1}{2}+\sqrt{\frac{1}{4}+k}\right)\cdot k}{\frac{1}{2}+\sqrt{\frac{1}{4}+k}+k} \end{aligned}$$

In order to simplify this equation further, we note that

$$\begin{aligned} \left(\frac{1}{2}+\sqrt{\frac{1}{4}+k}+k\right)\cdot\left(-\frac{1}{2}+\sqrt{\frac{1}{4}+k}\right) &= = -\frac{1}{4}+\left(\frac{1}{4}+k\right)+k\cdot\sqrt{\frac{1}{4}+k}-\frac{1}{2}k \\ &= \frac{1}{2}k+k\cdot\sqrt{\frac{1}{4}+k} \\ &= \left(\frac{1}{2}+\sqrt{\frac{1}{4}+k}\right)\cdot k \end{aligned}$$

Substitution of the above expression into the previous equation for C_1 then gives

$$\begin{aligned} C_1 &= W\cdot\frac{\left(\frac{1}{2}+\sqrt{\frac{1}{4}+k}+k\right)\cdot\left(-\frac{1}{2}+\sqrt{\frac{1}{4}+k}\right)}{\frac{1}{2}+\sqrt{\frac{1}{4}+k}+k} \\ &= W\cdot\left(-\frac{1}{2}+\sqrt{\frac{1}{4}+k}\right) \\ &= C_0 \end{aligned}$$

Hence, we are done. Provided that C_0 is given by 6, we have that $C_1 = C_0$ and more generally $C_t = C_{t-1}$. Therefore, we can say that $C_t = C_0$ for all t. This then means that the equivalent exponential smoothing parameter λ_{t-1} (see 5) is given by

$$\begin{aligned} \lambda_{t-1} &= \frac{C_0+W}{C_0+W+kW} \\ &= \frac{W\cdot\left(-\frac{1}{2}+\sqrt{\frac{1}{4}+k}\right)+W}{W\cdot\left(-\frac{1}{2}+\sqrt{\frac{1}{4}+k}\right)+W+kW} \\ &= \frac{\frac{1}{2}+\sqrt{\frac{1}{4}+k}}{\frac{1}{2}+\sqrt{\frac{1}{4}+k}+k} \end{aligned}$$

The table belows shows the resulting λ for a range of choices of k

k	λ
2	0.5
3.75	0.4
7.78	0.3
20	0.2
90	0.1
164.44	0.075
380	0.05
1560	0.025
9900	0.01

Therefore, for exponential smoothing with a λ of 0.05, for example, we would need $V = 380W$ (which implies a very high level of observation variance relative to the 1-step variance in the θ process).