

ISDA SIMM™,¹: From Principles to Model Specification

Counting Down to the Effective Date of the Rules

A. Introduction

The dawn of 2016 brings closer an important date for the non-centrally cleared OTC derivatives market, September 1st, the day when compliance with the margin requirement rules across jurisdictions will be required for the first time. The need to comply with these rules has triggered a holistic re-design of the collateral management, risk management, legal and reporting processes and systems within industry and is re-defining the modus operandi in this space. The global nature of the market, the degree of transformation that is required, the delay in the publication of the jurisdictional final rules and the compressed implementation timelines have all contributed to the creation of a unique Gordian knot that seeks a workable solution.

This paper will focus solely on the initial margin model that is proposed by ISDA, the Standard Initial Margin Model or SIMM, providing the context and rationale for the SIMM model specification. Currently, industry is working to refine, test, approve and validate the SIMM so that it can be ready to use by the rules' effective date.

B. Background

In December 2013, ISDA disclosed² the commencement of an industry initiative to develop a standard initial margin model that would be compliant with the BCBS-IOSCO guidelines³ and could be used by participants as a minimum for calling each other for initial margin (IM). This decision followed the realization that the OTC uncleared derivatives market could not operate viably under a schedule-based margin regime and that the development of a standardized, model-based IM methodology was both attainable and valuable, if adopted widely by firms.

A common methodology for IM quantification would have several key benefits, including the more efficient planning and management of firms' liquidity needs from margin calls, the timely and transparent dispute resolution as well as the consistent regulatory governance and oversight. In particular, the efficient resolution of disputes would be a considerable challenge if each participant developed its own IM model. If this were to occur, every firm would be compelled to build and maintain all of the IM models used by its trading partners, so that it could ascertain the correctness of the margin calls it receives. The operational complexity in the co-existence of a multitude of models and the capture of the relevant datasets for their implementation would overwhelm the industry and threaten the accomplishment of the regulatory objectives.

The first step in developing the SIMM methodically was to define the boundaries of the solution

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² ISDA, "ISDA Standard Initial Margin Model (SIMM™) for Non-Cleared Derivatives", Dec. 2013, [Link to ISDA website](#)

³ BCBS 261, "Margin requirements for non-centrally cleared derivatives", Sept. 2013, www.bis.org/publ/bcbs261.pdf

universe through articulating clearly the model's intended purpose, setting criteria for assessing candidate formulations and recognizing the modelling constraints imposed by the very nature of being a global and standardized IM model.

Objectives

The main objective stated in the final BCBS-IOSCO guidelines³ is the "reduction of systemic risk". Consequently, IM models are clearly differentiated from capital models, whose general aim is to accurately reflect all reasonable types of risk a portfolio may have. In essence, the global regulators recognized that a fine balance needs to be struck between risk sensitivity and the enhanced operational requirements of a margin model. Margin constitutes only one line of defense when a counterparty defaults, complemented by additional ones if it proves to be insufficient, and therefore focus has been given to capturing the systemically important risks of portfolios on an ongoing basis (risks that are not captured by the SIMM today but become systemically important in the future will then be incorporated).

In addition, the industry strongly believes that the value of the model is intrinsically linked to its market uptake and, as a result, its sophistication would need to be 'right-sized' to both comply with the regulatory requirements as well as be easy to understand and manage by market participants at large with varying levels of sophistication.

Criteria

ISDA identified the following key criteria to which an initial margin model aimed at satisfying the BCBS-IOSCO rules should adhere to.

- **Non-procyclicality** - Margins are not subject to continuous change due to changes in market volatility;
- **Ease of replication** - Easy to replicate calculations performed by a counterparty, given the same inputs and trade populations;
- **Transparency** - Calculation can provide contribution of different components to enable effective dispute resolution;
- **Quick to calculate** - Low analytical overhead to enable quick calculations and re-runs of calculations as needed by participants;
- **Extensible** - Methodology is conducive to addition of new risk factors and/or products as required by the industry and regulators;
- **Predictability** - IM demands need to be predictable to preserve consistency in pricing and to allow participants to allocate capital against trades;
- **Costs** - Reasonable operational costs and burden on industry, participants, and regulators;
- **Governance** - Recognizes appropriate roles and responsibilities between regulators and industry;
- **Margin appropriateness** - Use with large portfolios does not result in vast overstatements of risk. Recognition of risk factor offsets within the same asset class.

It is important to highlight that the SIMM is not a model that tries to optimize any particular dimension, instead finds a realistic and sustainable compromise between the criteria and the objectives that have been identified.

Modelling constraints

As initial margin calculations may involve the application of hundreds of shocks to the instrument, a full price re-evaluation calculation could take hours or even a whole day. On the other hand, it is imperative that the SIMM approximates the response to shocks with a fast calculation for derivative price-making decisions. The most efficient way to approximate a derivative contract's response to shocks is to pre-compute a sensitivity or "delta" of the derivative contract for each risk factor, and approximate the response by multiplying each sensitivity by the respective risk factor shock size.

Overall, the SIMM must remain relatively simple to apply while addressing the most serious systemic risks and avoiding high implementation costs for market users, so that market penetration is maximized and the disruption to this vital hedging market is minimized.

For additional details regarding the industry's views on issues raised in the "Background" section, please refer to the relevant publicly available ISDA paper².

C. Selecting the Model Specification

The ISDA WGMR Risk Classification & Methodology Workstream (ISDA RCM) was mandated to identify candidate IM models and then select the most suitable for SIMM. As a first step, the ISDA RCM investigated the merits and suitability of the existing banking capital models as well as the approaches used in the cleared derivatives field.

Scanning the existing industry solutions

In a capital model, one calculates the Expected Positive Exposure (EPE) to its counterparty in order to estimate the amount of credit risk capital to hold, given the counterparty's probability of default (PD). Regulatory counterparty exposure models are designed to calculate the EPE of derivative contracts traded with the counterparty, the credit risk capital is then estimated via the EPE, the PD of the counterparty, and the loss-given-default. However, unlike the risk mitigation provided by IM, the credit risk capital model requirement is imposed on the surviving counterparty and, consequently, the capital calculations need not be reconciled. Hence, capital model outputs do not require the same level of standardization as IM (though regulators may think otherwise so as to promote uniform financial safety). The ISDA RCM had to look beyond the traditional capital models for SIMM.

Looking at the cleared derivatives space and those IM models used by the major central counterparties (CCPs), we see that the co-existence of a number of models even within the same CCP's product coverage. Historical VaR simulations, the Standard Portfolio Analysis of Risk (SPAN) margin system and standardized approaches are all examples used by CCPs side-by-side. It seems that the underlying product risk characteristics drive different solutions with no model prevailing across the board. This finding confirms the complexity of selecting a unique SIMM specification and suggests that there is no single solution or approach.

SIMM Specification

A wide range of models was investigated, including factor-based parametric VaR models, historical simulation VaR models, use of risk grids/ladders and stable distribution methods. After a comprehensive evaluation of the model options, the ISDA RCM decided to base the SIMM on a variant of the Sensitivity Based Approach (SBA), an approach adopted by the BCBS for calculating capital requirements under the revised market risk framework; i.e. the Fundamental Review of the Trading Book (FRTB). SBA has been developed to be a more risk-sensitive yet conservative standard model for the market risk capital requirement quantification.

Although, the SIMM specification is still being refined and tested by the industry, its overall design has a number of distinct advantages that make it fit for purpose.

Non-procyclicality

A margin model that is procyclical is effectively flawed since it amplifies contagion and systemic risks when the financial marketplace is the most vulnerable; during a period of stress and high volatility. Certain models, such as historical simulation, have this feature embedded in their design and remedying it, within the regulatory bounds, can be quite challenging. The SIMM avoids this complication altogether; procyclicality only stems from the regulatory requirement to automatically recalibrate the model with certain frequency.

Data needs, costs and maintenance

The data needs, costs, accessibility and maintenance were paramount factors in the choice of the margin model specification. The SIMM is relatively parsimonious in its data requirements; it uses a “tiered” approach which first computes capital for various “buckets” using a standard Variance-Covariance formula, and then combines the bucket-level numbers using a modified Variance-Covariance formula which recognizes hedging and diversification. This avoids the need for a large covariance matrix covering all the risk factors, and keeps the calculation modular (which is helpful in reconciliation).

Furthermore, only the calibration agent (i.e. ISDA) needs to have access to certain historical timeseries for the SIMM parameter calibration (risk weights and correlations). The actual users do not need to have access to underlying raw data, avoiding the burden of any licensing costs. Having said that, the current SIMM calibration is mostly using data contributed from the ISDA member firms and hence avoids licensed data where possible.

In contrast, historical simulation and other approaches, would lead to elevated data usage costs and the need for a central authority to maintain and manage the full historical timeseries for the whole industry.

Transparency and Implementation costs

The identification of the drivers that impact the SIMM margin quantum is straightforward, enhancing the model’s predictability and facilitating internal communication by the users for liquidity and business planning purposes. At the same time, the SIMM calculator is simple to implement and cost-effective.

Despite its attractive features, the SIMM is still an approximate model that encompasses numerous compromises and simplifications with a view of i) satisfying the tight operational requirements of a cross-border international IM model and ii) making sure that any risk factors included can be actually reconciled across industry. For example, the need to calculate margin before quoting the price of a new trade is an important consideration, since margin has a direct impact on pricing through its funding cost. It was therefore essential to be able to perform the computation quickly not just for the current incremental IM requirement, but the expected future IM requirement through the life of the trade.

As a consequence, the SIMM has low granularity, simple assumptions in terms of distributions (Gaussian) and restricted risk coverage (e.g. dividend risk and interest rate skew are not captured). It is through backtesting and validation that assurance in the SIMM ability to cover the systemic risk of portfolios and adhere to the regulatory provisions is maintained. A standing ISDA committee (the ISDA SIMM Governance Committee) will review the results of the industry backtesting and approve any changes to the SIMM that are required to maintain regulatory compliance.

For additional information on the underlying mathematics of the SIMM, please refer to the Appendices A and B.

D. The Evolution of the SIMM throughout the regulatory process

The SIMM is a model that has evolved over time since its first release to regulators in September 2014. Industry testing, direct engagement with regulators and detailed requirements in the consultation papers and final rules released to-date in different jurisdictions have all contributed to shaping the SIMM. For example, the first version of the SIMM only captured the delta risk whereas jurisdictional rules subsequently specified that main non-linear dependencies should also be covered.

Nonetheless, until the rules are finalized in all major jurisdictions and the relevant competent authorities have the opportunity to review the SIMM, we can still expect some changes to be made to the model. Hopefully, these will have limited impact on firms' infrastructure builds and will not increase the pressure to meet the tight implementation timeline.

Throughout 2015 ISDA has been proactive in keeping the global regulators up-to-date with developments on the SIMM. As part of this engagement, ISDA developed and delivered to regulators complete model documentation, backtesting results and an independent model validation report.

ISDA remains committed to delivering a model that is compliant with the regulations at the major jurisdictions and is also looking ahead to the next phase, i.e. how the model will be governed after the effective date of September 1, 2016. Also, ISDA will support the industry as it faces implementation and compliance challenges in the coming years.

APPENDIX A: SIMM and the Nested VaR/CoVAR Formulas

1. Introduction

Both the FRTB Standardized Approach (Sensitivity Based Approach or SBA-C) and the ISDA SIMM use a sequence of nested variance/covariance formulas to calculate capital and margin.

The general form is to have a number of buckets a, b, c , etc., and a number of nodes $i = 1, \dots, n$ in each bucket. There is a risk-weighted delta WS_i^a for the delta to node i in bucket a .

The formulas for calculating the margin are as follows:

$$K_a^2 = \sum_{i=1}^n (WS_i^a)^2 + \sum_{i \neq j} \rho_{ij}^a (WS_i^a)(WS_j^a),$$

And

$$IM = \sqrt{\sum_a K_a^2 + \sum_{a \neq b} \gamma_{ab} S_a S_b},$$

Where S_a is a signed version of K_a which has different possible definitions

$$S_a = \begin{cases} \min\left(\max\left(-K_a, \sum_i WS_i^a\right), K_a\right), & \text{ISDA SIMM} \\ \sum_i WS_i^a, & \text{FRTB SBA-C} \end{cases}$$

However neither of these approaches gives a detailed justification of the nested formulas approach, so it is hard to judge which of the alternative formulas for S_a is more accurate.

2. Nested formulas justification

We can justify and motivate the nested formulas approach in the following way.

Let us define the random variables Y_i^a to be the random 10d evolution in the market rate corresponding to node i of bucket a . We assume that this has zero mean and unit variance, because the 10d scaling and 99% percentile have been put in the risk-weighted scaled delta. This lets us focus on the correlation structure.

Within each bucket a , the correlation structure of the nodes is given by a matrix U_a where

$$\text{Cov}(Y_i^a, Y_j^a) = (U_a)_{ij} = \rho_{ij}^a.$$

Let us denote the change in value of the portfolio due to changes in the market rate of node i of bucket a by X_{ai} , where

$$X_{ai} = WS_i^a Y_i^a.$$

So that this change is driven by the random variable Y_i^a which is the change in the relevant market rate.

Then the distribution of the change in value of the portfolio due to changes in bucket a over all its nodes is given by the formula

$$X_a = \sum_{i=1}^n X_{ai} = \sum_{i=1}^n WS_i^a Y_i^a.$$

The variance of this random variable is given by the formula

$$\text{Var}(X_a) = (WS^a)^\top U_a (WS^a) = K_a^2.$$

This shows, in line with our intuition, that K_a has a specific interpretation as the amount of PV variation caused by bucket a overall. So the first formula in the nested sequence makes sense.

The next nested formula is based on an idea of representing each overall bucket with an individual random variable. The random variable can be interpreted as the first principal component of changes in the bucket. For each bucket a , we have a random principal component Z_a , and we calibrate the covariance structure of these variables Z_a to have correlation γ_{ab} , where

$$\gamma_{ab} = \text{Cov}(Z_a, Z_b).$$

As before we have scaled the random variables to have unit variance.

We can derive an explicit formula for Z_a as follows. Let us denote the maximum eigenvalue of the correlation matrix U_a as λ_a , with corresponding eigenvector z_a , with unit length ($z_a^\top z_a = 1$). Then

$$Z_a = \lambda_a^{-1/2} \sum_{i=1}^n z_a[i] Y_i^a.$$

This has unit variance because $\text{Var}(Z_a) = \lambda_a^{-1} z_a^\top U_a z_a = 1$.

To derive the nested formula, we regress the random variable X_a against the bucket's principal component Z_a , to write it as a multiple of Z_a plus an independent term ε_a . That is we write (with no approximation)

$$X_a = S_a Z_a + \varepsilon_a, \quad \text{where} \quad S_a = \text{Cov}(X_a, Z_a).$$

We assume the correlation structure that the ε_a are independent of both each other and the principal components.

The variance of ε_a is given by the formula $\text{Var}(\varepsilon_a) = K_a^2 - S_a^2$. This will always be non-negative, because of the Cauchy-Schwarz inequality

$$|S_a| = |\text{Cov}(X_a, Z_a)| \leq \text{Var}(X_a)^{1/2} \text{Var}(Z_a)^{1/2} = K_a.$$

Then the total portfolio value change X will be given by

$$X = \sum_a X_a = \sum_a \varepsilon_a + \sum_a S_a Z_a.$$

Its variance is the square of the total margin requirement and is equal to

$$IM^2 = Var(X) = \sum_a Var(\varepsilon_a) + \sum_a S_a^2 + \sum_{a \neq b} \gamma_{ab} S_a S_b.$$

We can substitute $Var(\varepsilon_a) = K_a^2 - S_a^2$ into the above to get

$$IM^2 = \sum_a K_a^2 + \sum_{a \neq b} \gamma_{ab} S_a S_b.$$

This is the nested variance/covariance formula as required.

3. Explicit formula for S_a

We can now derive the actual explicit formula for S_a . The formulas used by both FRTB and SIMM are approximations to the true value.

Recall from above that S_a is the covariance between the risk vector X_a and the bucket's principal component Z_a . The covariance can be written as

$$\begin{aligned} S_a &= Cov(X_a, Z_a) = Cov\left(\sum_{i=1}^n WS_i^a Y_i^a, \lambda_a^{-1/2} \sum_{i=1}^n z_a[i] Y_i^a\right) \\ &= \lambda_a^{-1/2} (WS^a)^T U_a z_a = \lambda_a^{1/2} (WS^a)^T z_a. \end{aligned}$$

3.1 FRTB approximation

The FRTB methodology makes the approximation $S_a = \sum WS_i^a$. This is equivalent to the eigenvector z_a being constant and its eigenvalue $\lambda_a = \sqrt{n}$. This only happens if every correlation ρ_{ij}^a is exactly one. Otherwise this approximation is not exact.

It also has the drawback that this approximation for S_a can exceed K_a , which is impossible in reality. This could cause an erroneous over-estimation of the capital.

3.2 SIMM approximation

The SIMM methodology makes a different approximation:

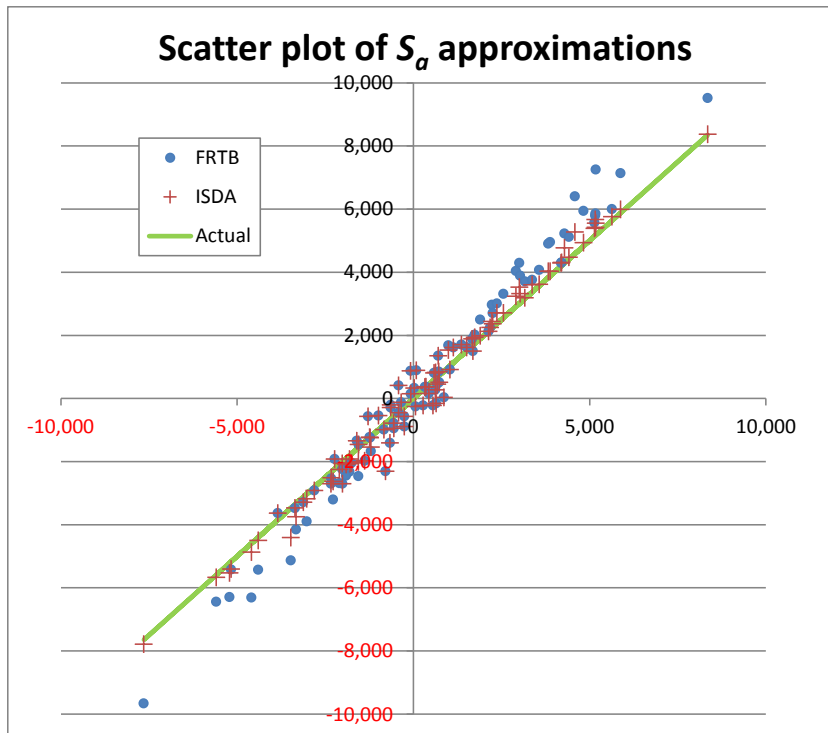
$$S_a = \min\left(\max\left(-K_a, \sum_i WS_i^a\right), K_a\right).$$

This has the advantage that it cannot go outside the allowed bounds of $\pm K_a$, but it is still only an approximation.

3.3 Testing the approximations

We can test the approximations by calculating the true value of S_a and also calculating the values of the FRTB approximation and the ISDA approximation.

We did this for a 100 random samples of possible risk vectors for an interest-rate bucket. There are 10 nodes in the bucket (3m, 6m, 1y, 2y, 3y, 5y, 10y, 15y, 20y, 30y), and each node had a risk delta which was an independent normal random variable, with a standard deviation of USD 1,000.



The scatter plot shows the 100 samples. For each sample, there are two markers: a blue dot for the FRTB approximation and a red cross for the ISDA approximation. The green line marks the actual value of S_a calculated using the covariance formula. Values above the green line indicate an approximation which is too high, and values below the green line indicate an approximation which is too low. The x co-ordinate of the markers is the true value, and the y co-ordinate is the approximation.

The graph shows that generally the ISDA approximation is closer to the actual value than the FRTB approximation. In many cases, near zero, the two approximations are the same. In numerical terms the ISDA approximation has an average error USD 350, but the FRTB approximation has an average error of USD 550. So the FRTB approximation is about 50% less accurate than the ISDA approximation.

The ISDA approximation is preferred over the analytic formula because it is robust and easier to calculate and reconcile between firms.

4. Explicit large correlation matrix

This interpretation also allows us to calculate the explicit large correlation matrix which is effectively being used to calculate SIMM.

We can show the construction explicitly in the two basket case, where the baskets are called a and b . Let us diagonalise the correlation matrices U_a and U_b so that (in the case of a)

$$U_a = P_a \Lambda_a P_a^T,$$

where P_a is an orthogonal matrix of the eigenvectors of U_a and Λ_a is the diagonal matrix of eigenvalues. We have sorted the eigenvalues so that the first eigenvalue is the largest one (denoted λ_a). We define a "square root" matrix

$$V_a = P_a \Lambda_a^{1/2},$$

so that

$$V_a V_a^T = U_a \quad \text{and} \quad V_a^{-1} U_a (V_a^{-1})^T = I.$$

Then we can create the principal component random vectors

$$R_a = V_a^{-1} Y^a.$$

This is a normal random vector with zero mean and covariance matrix of $V_a^{-1} U_a (V_a^{-1})^T = I$, which is the identity matrix. Our correlation structure is that the first element of R_a and the first element of R_b have correlation of γ_{ab} , but the other elements of the R-vectors are independent. This corresponds to the intuitive sense that the first principal components of different buckets are correlated, but there is no correlation between the secondary principal components.

So the covariance matrix of the combined vector of both R_a and R_b is

$$\text{Covar} \begin{pmatrix} R_a \\ R_b \end{pmatrix} = \begin{pmatrix} I & \gamma_{ab} \Delta_{11} \\ \gamma_{ab} \Delta_{11} & I \end{pmatrix},$$

where Δ_{11} is the matrix which has all zero entries except for the top-left cell which is one.

Since we can express the original Y-vectors in terms of the R-vectors, as

$$\begin{pmatrix} Y^a \\ Y^b \end{pmatrix} = \begin{pmatrix} V_a & 0 \\ 0 & V_b \end{pmatrix} \begin{pmatrix} R_a \\ R_b \end{pmatrix},$$

so that the covariance of the Y-vectors is given by

$$\text{Covar} \begin{pmatrix} Y^a \\ Y^b \end{pmatrix} = \begin{pmatrix} V_a & 0 \\ 0 & V_b \end{pmatrix} \begin{pmatrix} I & \gamma_{ab} \Delta_{11} \\ \gamma_{ab} \Delta_{11} & I \end{pmatrix} \begin{pmatrix} V_a^T & 0 \\ 0 & V_b^T \end{pmatrix} = \begin{pmatrix} U_a & \gamma_{ab} V_a \Delta_{11} V_b^T \\ \gamma_{ab} V_b \Delta_{11} V_a^T & U_b \end{pmatrix}.$$

If we define the scaled eigenvector $y_a = \lambda_a^{1/2} z_a$, where z_a is the unit-length eigenvector corresponding to the maximum eigenvalue λ_a , then the covariance matrix can also be written as:

$$\text{Covar} \begin{pmatrix} Y^a \\ Y^b \end{pmatrix} = \begin{pmatrix} U_a & \gamma_{ab} y_a y_b^T \\ \gamma_{ab} y_b y_a^T & U_b \end{pmatrix}.$$

5. Numerical Example for GIRR

We can see numerical values for these vector and matrix quantities in the case of the SIMM calibration for GIRR. Let us ignore tenor basis and just focus on the buckets (currency curves) and nodes (tenor points).

The correlation matrix is a 10 x 10 matrix over the ten tenor points.

The maximum eigenvalue is 7.243, and the scaled eigenvector ($y_a = \lambda_a^{1/2} z_a$) has the values

3m	6m	1y	2y	3y	5y	10y	15y	20y	30y
0.474	0.68	0.833	0.915	0.948	0.967	0.936	0.906	0.885	0.844

So the true S_a is calculated by calculating the weighted sum of WS_i^a , weighted by this eigenvector y_a .

Then the off-diagonal block matrix, let's call it A , is $A = V_a \Delta_{11} V_b^T = y_a y_b^T$, has values

	3m	6m	1y	2y	3y	5y	10y	15y	20y	30y
3m	0.225	0.322	0.395	0.434	0.450	0.458	0.444	0.429	0.420	0.400
6m	0.322	0.462	0.566	0.622	0.645	0.657	0.636	0.615	0.602	0.574
1y	0.395	0.566	0.694	0.762	0.790	0.805	0.780	0.754	0.738	0.703
2y	0.434	0.622	0.762	0.837	0.868	0.884	0.856	0.828	0.810	0.772
3y	0.450	0.645	0.790	0.868	0.899	0.917	0.888	0.859	0.840	0.800
5y	0.458	0.657	0.805	0.884	0.917	0.935	0.905	0.876	0.856	0.816
10y	0.444	0.636	0.780	0.856	0.888	0.905	0.876	0.848	0.829	0.790
15y	0.429	0.615	0.754	0.828	0.859	0.876	0.848	0.820	0.802	0.764
20y	0.420	0.602	0.738	0.810	0.840	0.856	0.829	0.802	0.784	0.747
30y	0.400	0.574	0.703	0.772	0.800	0.816	0.790	0.764	0.747	0.712

So the covariance matrix of the two currency vectors together is (in block form)

$$Covar = \begin{pmatrix} U & \gamma_{ab} A \\ \gamma_{ab} A & U \end{pmatrix}.$$

Note that there is no subscript on the intra-bucket correlation U matrices, because they are the same for each currency. The value of γ_{ab} is [27]% with the current calibration.

Only in the very special case where the correlation matrix U has all entries equal to one, will the matrix A also have all entries equal to one.

In all other cases, A has entries less than or equal to one, because the entries of y_a are all bounded by one in modulus (see appendix for proof).

Supplement – Proof that elements of scaled eigenvectors are smaller than 1 in magnitude

Suppose the orthonormal eigenvectors of a correlation matrix U are z_1, z_2, \dots, z_n with eigenvalues $\lambda_1, \lambda_2, \dots, \lambda_n$. Then the matrix U can be written as

$$U = \sum_{i=1}^n \lambda_i z_i z_i^T.$$

Since the diagonal entries of U all have the value one, then

$$1 = \sum_{i=1}^n \lambda_i z_i^2[k], \quad \text{for each } k = 1, \dots, n.$$

Define the scaled eigenvector $y_i = \lambda_i^{1/2} z_i$, then

$$1 = \sum_{i=1}^n y_i^2[k], \quad \text{for each } k = 1, \dots, n.$$

Thus we can deduce that each co-ordinate of each y -vector is bounded above in magnitude by one.

APPENDIX B: SIMM Curvature Formulas

1. Introduction

In previous versions of SIMM (prior to v3.4), the curvature margin was modeled using the similar methodology from FRTB.

$$K_b = \sqrt{\max\left(0, \sum_k \max(CVR_k, 0)^2 + \sum_k \sum_{l \neq k} \rho_{kl} CVR_k CVR_l \psi(CVR_k, CVR_l)\right)}$$

$$CurvatureMargin = \sqrt{\max\left(0, \sum_b K_b^2 + \sum_b \sum_{c \neq b} \gamma_{bc} S_b S_c \psi(S_b, S_c)\right)} + K_{residual}$$

During SIMM back testing of delta-neutral portfolios, and found that quite a few portfolios failed the back testing. Then a simple and straightforward proposal to use the same aggregation structure as Delta margin was tested

$$K = \sqrt{\sum_k CVR_k^2 + \sum_k \sum_{l \neq k} \rho_{kl} CVR_k CVR_l}$$

$$CurvatureMargin = \sqrt{\sum_b K_b^2 + \sum_b \sum_{c \neq b} \gamma_{bc} S_b S_c} + K_{residual}$$

It also failed the back testing. The fundamental cause behind the failure is that the curvature term is essentially chi-squared in nature, but both FRTB and Delta approach are based on the normal distribution.

For a portfolio with both linear and curvature risks, the 10-day (10 business days) PL can be written as

$$PL = \mathbf{\Delta}^T \cdot \delta \mathbf{X} + \frac{1}{2} \delta \mathbf{X}^T \cdot \mathbf{\Gamma} \cdot \delta \mathbf{X}$$

Where $\mathbf{\Delta}$ is a vector of all linear risks (Deltas), $\mathbf{\Gamma}$ is a matrix of Gamma, and $\delta \mathbf{X}$ is a vector of 10-day move of all market factors. Define the covariance matrix of $\delta \mathbf{X}$ as $\mathbf{\Sigma}$. Using moment matching, the VaR can be written as the following form:

$$VaR = \frac{1}{2} \text{Tr}(\mathbf{\Gamma} \mathbf{\Sigma}) + Z_{CF} \sqrt{\mathbf{\Delta}^T \mathbf{\Sigma} \mathbf{\Delta} + \frac{1}{2} \text{Tr}(\mathbf{\Gamma} \mathbf{\Sigma})^2},$$

where Z_{CF} can be estimated using Cornish-Fisher expansion, and the zero-order is $Z_{CF} = \Phi^{-1}(99\%) = 2.33$.

In ISDA SIMM model, the margin requirements from Delta and Curvatures are separately calculated and then added together. In order to calculate the curvature margin requirement, we consider a portfolio with zero delta risks. The curvature SIMM requirement is

$$VaR = \frac{1}{2} \text{Tr}(\Gamma \Sigma) + Z_{CF} \sqrt{\frac{1}{2} \text{Tr}(\Gamma \Sigma)^2}$$

If there is no cross Gamma, the above formula can be simplified as

$$VaR = \frac{1}{2} \sum \Gamma_k \left(\frac{RW_k}{\Phi^{-1}(99\%)} \right)^2 + Z_{CF} \sqrt{\frac{1}{2} \sum \rho_{lk}^2 \Gamma_l \left(\frac{RW_l}{\Phi^{-1}(99\%)} \right)^2 \Gamma_k \left(\frac{RW_k}{\Phi^{-1}(99\%)} \right)^2}$$

Where RW_k are the risk weights which have been calibrated to the 99% percentile of historical 10-day market movements. So $\frac{RW_k}{\Phi^{-1}(99\%)}$ are the historical 10-day standard deviations.

2. ISDA SIMM Curvature Formulas

In SIMM, we calculate Gamma by using the Vega-Gamma relationship. This can be written as

$$\Gamma_k = \frac{1}{\sigma_k^2 \cdot \frac{t \text{ days}}{365}} \cdot \sigma_k \frac{\partial V}{\partial \sigma_k}$$

where σ is the implied volatility and t is the expiry time of the option. The curvature risk exposures are defined as

$$CVR_k = \frac{1}{2} \Gamma_k \left(\frac{RW_k}{\Phi^{-1}(99\%)} \right)^2 = \frac{1}{2} \sigma_k \frac{\partial V}{\partial \sigma_k} \frac{14}{t \text{ days}} \left(\frac{RW_k}{\Phi^{-1}(99\%) \sigma_k \sqrt{\frac{14}{365}}} \right)^2$$

We realized that it is difficult for firms to get all implied volatilities for SIMM calculations, and we further assumed that the implied volatilities can be approximated from risk weights. The above formula can be simplified to

$$CVR_k = \frac{1}{2} \sigma_k \frac{\partial V}{\partial \sigma_k} \frac{14}{t \text{ days}}$$

Then for each bucket b and risk factor k with multiple tenors, we have

$$CVR_{b,k} = \sum_{i \text{ in } b} \sum_j SF(t_{kj}) \cdot \sigma_{kj} \frac{\partial V_i}{\partial \sigma}, \text{ where } SF(t) = 0.5 \min \left(1, \frac{14 \text{ days}}{t \text{ days}} \right).$$

The sum is over all vol-tenor j . Using the definition of CVR, we can write the Curvature margin formula as

$$CurvatureMargin = \sum_m CVR_m + \lambda \sqrt{\left[\sum_m CVR_m^2 + \sum_{n \neq m} \rho_{mn}^2 CVR_m CVR_n \right]}$$

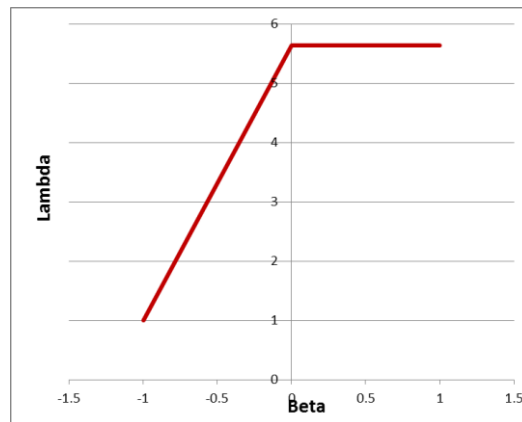
Where $\lambda = \sqrt{2}Z_{CF}$. The λ expression is an interpolation between two known edge cases. It is an approximation formula which will be close to or slightly more conservative than the actual values. Let us define

$$\beta = \frac{\sum_{b,k} CVR_{b,k}}{\sum_{b,k} |CVR_{b,k}|}$$

We require λ as a function of β with the following properties:

1. Consider the single-bucket single-risk-factor case. If $CVR_1 = X > 0$, then the PnL has a Chi-square distribution, and the 99% percentile is approximately equal to $\Phi^{-1}(0.995)^2 X$. So if $\beta = 1$, we want $\lambda = \Phi^{-1}(0.995)^2 - 1$. But if $CVR_1 = X < 0$, equivalently $\beta = -1$, then the curvature term is non-positive, so a conservative value for it is zero, which is given by $\lambda = 1$.
2. In the more general case, of a portfolio in which each trade has negative Gamma (so $\beta = -1$), we also want the curvature margin to be zero. The condition $\lambda = 1$ is sufficient for this.
3. For negative β , we choose to have λ as an increasing function which reaches its maximum when $\beta = 0$.

A simple form of such function can be piece-wise linear, as follows



We define

$$\theta = \min(\beta, 0)$$

The above function of λ will be represented as the following formula

$$\lambda = (\Phi^{-1}(99.5\%)^2 - 1)(1 + \theta) - \theta$$

Thus we obtain:

$$CurvatureMargin = \max\left(\sum_{b,k} CVR_{b,k} + \lambda \sqrt{\sum_{b,k} CVR_{b,k}^2 + \sum_{(b,k) \neq (c,l)} U_{bk,cl}^2 CVR_{b,k} CVR_{c,l}}, 0\right),$$

where the correlation term is

$$U_{bk,cl} = \begin{cases} = \rho_{kl} & \text{for } b = c \\ \cong \gamma_{bc} & \text{for } b \neq c \end{cases}$$

For $b \neq c$, γ_{bc} is an approximation of the real correlation term (see Appendix A for more details).

With this we can rewrite the equation as:

$$CurvatureMargin = \max \left(\sum_{b,k} CVR_{b,k} + \lambda \sqrt{\sum_{b,k} CVR_{b,k}^2 + \sum_{b,k \neq l} \rho_{kl}^2 CVR_{b,k} CVR_{b,l} + \sum_{b \neq c} \sum_{k,l} \gamma_{bc}^2 CVR_{b,k} CVR_{c,l}}, 0 \right),$$

$$CurvatureMargin = \max \left(\sum_{b,k} CVR_{b,k} + \lambda \sqrt{\sum_b K_b^2 + \sum_{b \neq c} \gamma_{bc}^2 \left(\sum_k CVR_{b,k} \right) \left(\sum_l CVR_{c,l} \right)}, 0 \right),$$

where

$$K_b = \sqrt{\sum_k CVR_{b,k}^2 + \sum_{k \neq l} \rho_{kl}^2 CVR_{b,k} CVR_{b,l}}$$

is the curvature risk exposure aggregated for each bucket b .

As $K_b^2 + \sum_{b \neq c} \gamma_{bc}^2 \left(\sum_k CVR_{b,k} \right) \left(\sum_l CVR_{c,l} \right)$ can be negative in some cases, we set (as we usually do with SIMM in that case) $S_b = \max(\min(\sum_k CVR_{b,k}, K_b), -K_b)$ for each bucket b and finally obtain

$$CurvatureMargin \cong \max \left(\sum_{b,k} CVR_{b,k} + \lambda \sqrt{\sum_b K_b^2 + \sum_{b \neq c} \gamma_{bc}^2 S_b S_c}, 0 \right).$$

3. Numerical Tests

We setup a set of testing portfolios, which have only curvature components. The PL can be simulated using

$$PL = \sum_{i=1}^N CVR_i \epsilon_i^2$$

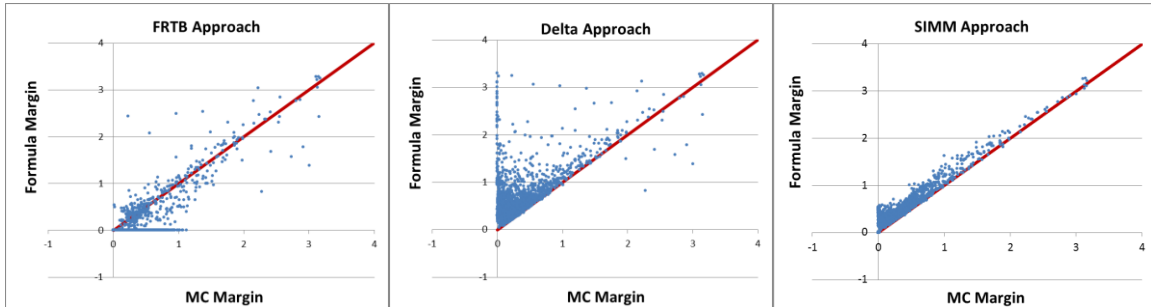
Where ϵ_i are correlated standard normal random variables. We further assume that

- The correlations between all pairs are the same.
- The sum of absolute CVRs of all dimensions is equal to 1. The SIMM margins for all testing portfolios are in the same order and so it is easy to compare.

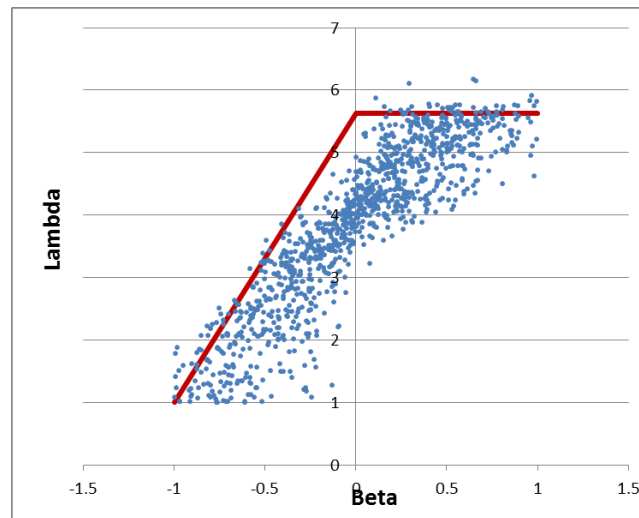
We set up different portfolios that

- Number of dimensions are from 2 to 1024
- Correlations are from -1 to 1
- β are from -1 to 1

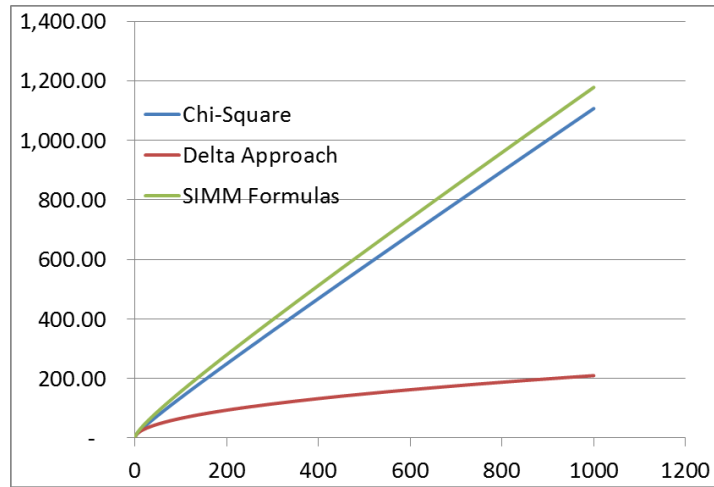
The VaR can be calculated more accurately using Monte Carlo simulation. In the tests, we have used 10,000 paths for each Monte Carlo simulation. The following graphs show the comparisons between Monte Carlo simulation and the margin formulas.



We have compared λ with Monte Carlo simulated actual values (blue dots in the following graph) to the proposed simple formula. It demonstrates that the simple function of λ is almost always conservative (dots are below the red line).



In addition to above Monte Carlo tests, we have tested SIMM formulas in a special case where an analytical test can be done. When there are n un-correlated underline market factors with the same positive gamma: $CVR_i = X > 0$, the actual 99% percentile can be calculated using the Chi-Square table. The following graph shows the 99% percentile of the exact value from the Chi-Square table, Delta-Approach, and current SIMM formulas. The x-axis is “ n ” — the number of variables, and the y-axis is the 99% percentile.



From all above tests, we have demonstrated that SIMM formulas captured curvature risks pretty well, and are slightly more conservative.